9. A pizza is in the shape of a square whose diagonal measures $3\sqrt{2\pi}$ inches. Find the diameter of the circular pizza with the same area.

ANSWER: 6 **(The side of the square is $3\sqrt{\pi}$, so the area is 9π. A circle with radius 3 has the same area. Thus, the diameter of the circle is 6.)**

10. Suppose that $\sin(2\alpha) = -\frac{3}{5}$, and $\cos(2\alpha) = \frac{4}{5}$. What is the value of $\cot(\alpha)$?

ANSWER: -3 $(\cot(\alpha) = \frac{\sin(2\alpha)}{1-\cos(2\alpha)})$

11. What is the algebraic sum of all solutions to the following equation?

$$(x^3 - 1)(2x - 1)(4x + 3)(x + 1)(x^2 - 4x + 2) = 0$$

ANSWER: $\frac{11}{4} = 2.75 = 2\frac{3}{4}$ **(The sum of all roots of $x^3 - 1$ is 0. The roots of the next three factors are $\frac{1}{2}$, $-\frac{3}{4}$, and -1. The sum of the roots of $(x^2 - 4x + 2)$ is 4. So the sum of all solutions of the equation is $\frac{11}{4}$.)**

12. Sonya has made 5 of 17 free throws. How many consecutive free throws must she make from this point forward to raise her percentage of successful free throws to 75%?

ANSWER: 31 **(She has missed 12 free throws. To bring her shooting percentage to 75%, she must make a total of 36. As she has already made 5, she must now make 31 consecutive shots.)**

13. Maria and Lisa play a game. At the end of each game, the loser gives the winner a quarter. Maria has won three games, and Lisa has 4 more quarters than she did when she began. How many games did they play?

ANSWER: 10 **(Lisa has won 7 games, and Maria has won 3.)**

14. Define $f(x) = 4x - x^2$ and suppose that $x_1 = 2 - \sqrt{3}$. If $x_{n+1} = f(x_n)$ for all $n \geq 1$, what is x_{2008}?

ANSWER: 3 $(x_2 = f(x_1) = 1, x_3 = f(x_2) = 3, x_4 = f(x_3) = 3, \ldots)$

15. What is the number of digits in $4^{24}5^{41}$ when written in the usual base ten form?

ANSWER: 44 $(4^{24}5^{41} = 2^{48}5^{41} = 2^7 2^{41}5^{41} = 128(10^{41}))$

16. Bill was overheard making the following two statements. Both were spoken on the same day and both were true. Two days ago I was 14. Next year I'll be 17. On what date is Bill's birthday? (The year is not required.)

ANSWER: December 31 **(The statement was made on January 1 the day after Bill's 15th birthday. Bill will be 16 later in the year the statement is made and will be 17 in the following year.)**

1. How many lines of symmetry does a regular octagon have?

   ANSWER: 8 **(4 through pairs of opposite vertices and 4 through pairs of midpoints of opposite sides)**

2. What is $\arccos(\cos(-\frac{3\pi}{4})) + \arcsin(\sin(\frac{7\pi}{12}))$?

   ANSWER: $\frac{7\pi}{6} = \frac{14\pi}{12}$ $(= \frac{3\pi}{4} + \frac{5\pi}{12})$

3. $P(x)$ is a polynomial function of degree 8 defined as follows. $P(x) = (x^2 - 5)^2(x^2 + 3)(x^2 - 7x + 13)$. How many real solutions are there to the equation $P(x) = 0$?

   ANSWER: 2 **(The first factor has two distinct real solutions and neither of the other factors has any.)**

4. A number X has two digits a and b. The difference between a and b is 3 and the number X is seven times the sum of a and b. What is X?

   ANSWER: 63 $(6 - 3 = 3$ **and** $63 = 7(6 + 3))$

5. ABCD is a rectangle with an area of 36 square units. Points E, F, and G are midpoints of three sides of the rectangle. The area of the triangle EFG is how many square units?

   ANSWER: 9 **(No matter which sides are used, the triangle covers exactly half of one half of the rectangle.)**

6. Determine all values of x that satisfy the following equation.

$$x^2 \cdot 27^{x/3} - x \cdot 3^{x+2} - 10 \cdot 3^x = 0$$

   ANSWER: $x = 10$, $x = -1$

   (The left-hand-side factors as $3^x(x^2 - 9x - 10) = 3^x(x - 10)(x + 1)$**.)**

7. What value must c have so that the following expression is a perfect square?

$$\frac{1}{4}x^2 - \frac{2}{3}x + (1 + c)$$

   ANSWER: $c = -\frac{5}{9}$ $((\frac{1}{2}x - \frac{2}{3})^2 = \frac{1}{4}x^2 - \frac{2}{3}x + (1 + -\frac{5}{9}))$

8. Jay walks from point A to point B by following the line segments in the grid shown. If he always walks East (to the right in the grid) or North (upward in the grid), how many different routes can he take from A to B?

   (B is 5 segments east and 3 segments north of A.)

   ANSWER: 56 **(In choosing the path he takes, Jay must choose 5 eastbound segments in the 8 blocks he walks. The remainder are northbound. So, the answer is** $\binom{8}{5} = \frac{8!}{5!3!} = 56$**.)**