INSTRUCTIONS: This is a 90-minute, 40-problem, multiple-choice examination. There are five (5) possible responses to each question or problem. You are to select the one (1) “best” answer to each. You may mark on the test booklet, and the back of each page may be used for additional work space. When you are sure of your answer, circle the letter of the choice you have made on the test booklet. After you have worked all problems you can work, transfer your answers to the score sheet which has your student number encoded. Darken completely the blank below the letter of your response to each question. Mark your answers boldly with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark below row 40. Your score will be computed by the formula 40 + (4C - I), where C represents the number of correct answers and I represents the number of incorrect answers. If you can definitely rule out at least one choice it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. You may not use a calculator on this test.

Review and check your score sheet carefully. Your student identification number has been encoded on your red and white score sheet and it has been checked by our marked-sense card reader. This number is in the “I.D. Number” section at the top; if you alter this number in any way you may disqualify yourself and your team from consideration for any awards.

When you complete your test, bring your answer sheet to the Test Monitor. You may keep your pencil and test booklet. You may leave the room after you have handed in your answer sheet.

PLEASE DO NOT OPEN
UNTIL INSTRUCTED TO DO SO
1. Farmer Williams sells apples and oranges at his roadside stand. Apples are $0.30 each or $3.00 a dozen. Oranges are $0.40 each or $4.00 a dozen. Which of the following would cost the least?

A. Six dozen apples and five dozen oranges
B. Five dozen apples and six dozen oranges
C. Four dozen apples and seven dozen oranges
D. Eleven dozen apples and one dozen oranges
E. You can't compare apples and oranges.

2. What does \( \left( \frac{1}{4} \right)^2 + \sqrt{\frac{3}{2} - \sqrt{2}} \) equal?

A. \(-4\)
B. \(4\)
C. \(\frac{1}{4}\)
D. \(-\frac{1}{4}\)
E. 5

3. A certain circle has area equal to 1. What is its diameter?

A. \(1\)
B. \(\sqrt{\pi}\)
C. \(2\sqrt{\pi}\)
D. \(\frac{\pi}{2}\)
E. \(\frac{2\sqrt{\pi}}{\pi}\)

4. A square has sides of length 2. Each side of the square is used as the diameter of a circle as in the diagram. What is the shaded area?

A. \(4 - \pi\)
B. \(8 - 2\pi\)
C. \(\pi - 2\)
D. \(2\pi - 4\)
E. Correct answer not given
5. Line \( L_1 \) is parallel to line \( L_2 \). Line \( L_3 \) is perpendicular to line \( L_4 \). \( \alpha = 20^\circ \). What is \( \beta \)?

A. 70°
B. 120°
C. 150°
D. 160°
E. Correct answer not given

6. Consider one diagonal of a 4 by 5 rectangle. What is the largest angle formed by this diagonal and one of the rectangle's sides?

A. \( \tan^{-1}\left(\frac{4}{5}\right) \)
B. \( \sec^{-1}\left(\frac{\sqrt{41}}{4}\right) \)
C. \( \sin^{-1}\left(\frac{4}{5}\right) \)
D. \( \cos^{-1}\left(\frac{5}{4}\right) \)
E. \( \csc^{-1}\left(\sqrt{41}\right) \)

7. How many diagonals does a convex octagon have? (A diagonal is a line segment connecting nonadjacent vertices of the octagon.)

A. 4
B. 10
C. 20
D. 28
E. Correct answer not given

8. If \( f(x) = 2x + 3 \) and \( g(x) = x^2 + 2 \), find the sum of the solutions of \( (f \circ g)(x) = (g \circ f)(x) \).

A. 6
B. 5
C. -1
D. -5
E. -6

9. If \( f(x) = \frac{6}{x^2 - 1} + \frac{2}{x^2 + 1} = \frac{3}{x^2 + 1} \) have?
9. \( \frac{\sqrt[5]{x^{-3}} \cdot 3\sqrt[3]{x^{15}}}{x^{3t}} \) equals which of the following?

A. \( x^7 \)
B. \( x^{10} \)
C. \( x^{13} \)
D. \( x^{14} \)
E. \( x^7 \)

10. Which is largest?

A. \( \sin(50°) \)
B. \( \cos(50°) \)
C. \( \tan(50°) \)
D. \( \cot(50°) \)
E. \( \csc(-50°) \)

11. Which of the following is the directrix of \( y^2 + 6y + 8x = -41 \)?

A. \( x = -2 \)
B. \( x = -6 \)
C. \( y = -1 \)
D. \( y = -5 \)
E. Correct answer not given

12. \( A \) is the largest solution and \( B \) is the smallest solution of \( x^2 + x - 3 = 0 \). What is \( 2A - 4B \)?

A. \( 1 + 3\sqrt{13} \)
B. \( 1 - \sqrt{13} \)
C. \( -1 - 3\sqrt{13} \)
D. \( -1 + \sqrt{13} \)
E. Correct answer not given

13. How many distinct real solutions does \( \frac{6}{x^4 - 1} + \frac{2}{x^2 + 1} = \frac{3}{x^2 - 1} \) have?

A. 0
B. 1
C. 2
D. 3
E. 4
14. \( \frac{8y^3 + 27}{4y^2 + 12y + 9} + \frac{12y^2 - 18y + 27}{6y^2 + 9y} \) is which of the following?

A. \( \frac{y(2y + 3)^2}{2y^2 - 6y + 9} \)
B. \( \frac{y(4y^2 - 12y + 9)}{12y^2 - 6y + 9} \)
C. \( 3y \)
D. \( 3 \)
E. \( y \)

15. Which of the following is the equation of the circle centered at (-1,2) tangent to \( y = x - 2 \)?

A. \( 4x^2 + 4y^2 - 8x + 16y = 50 \)
B. \( 4x^2 + 4y^2 + 8x - 16y = 50 \)
C. \( 4x^2 + 4y^2 - 8x + 16y = 30 \)
D. \( 4x^2 + 4y^2 + 8x + 16y = 30 \)
E. \( 4x^2 + 4y^2 + 8x - 16y = 30 \)

16. If \( x^4 - 16y^4 = -175 \) and \( x^3 + 2x^3y + 4xy^2 + 8y^3 = 175 \), find \( x - 2y \).

A. \(-1\)
B. \(1\)
C. \(-25\)
D. \(25\)
E. Correct answer not given

17. What is the ones digit of \( 1! + 2! + 3! + 4! + \cdots + 100! \)?

A. 0
B. 1
C. 2
D. 3
E. 4

18. A, B, C, D are distinct points on the circle. What is \( m (< ACB) + m (< DBA) + m (< CAD) + m (< BDC) \)?

A. \( 90^\circ \)
B. \( 180^\circ \)
C. \( 360^\circ \)
D. It depends on the placement of the points.
E. Correct answer not given
19. What is \( \frac{f(x + h) - f(x)}{h} \) if \( f(x) = 3x^2 - 6x - 2 \)?

A. \( 3x + 3h - 6 \)
B. \( 3h - 6 \)
C. \( 1 \)
D. \( 6x + 3h + 6 \)
E. \( 6x + 3h - 6 \)

20. Which of the following is the largest y-coordinate of a point on an asymptote of \( y^2 - 4x^2 + 2y - 16x = 19 \) when \( x = 5 \)?

A. 11
B. 12
C. 13
D. 14
E. 15

21. Solve for \( x \):
\[
\frac{3x - 5}{2} = \frac{-2(x + 4)}{3} - \frac{7}{6}.
\]

A. \( -\frac{8}{13} \)
B. \( \frac{8}{13} \)
C. \( -\frac{18}{8} \)
D. \( \frac{18}{8} \)
E. \( \frac{23}{13} \)

22. A stationary state patrol officer is observing traffic going east on an east-west interstate. Looking due north he spots a red sports car from a distance of 1,000 feet. At which time he starts a timer. Four seconds later after turning 20° to the east he again spots the car. Which of the following is closest to the speed of the car? \( \sin(20°) = .342 \), \( \cos(20°) = .940 \), \( \tan(20°) = .364 \), 60 mph = 88 ft/sec.

A. 62 mph
B. 74 mph
C. 85 mph
D. 50 mph
E. 80 mph
23. If the graph of \( y = |x| \) is shifted 2 units to the left and 3 units up, what is its equation?
   A. \( y = |x - 2| + 3 \)
   B. \( y = |x + 2| + 3 \)
   C. \( y = |x - 2| - 3 \)
   D. \( y = |x + 2| - 3 \)
   E. Correct answer not given

24. Simplify \( \tan(\theta) + \frac{\cos(\theta)}{1 + \sin(\theta)} \).
   A. 1
   B. 0
   C. \( \sin(\theta) \)
   D. \( \cos(\theta) \)
   E. \( \sec(\theta) \)

25. Four friends go fishing one day and bring home a total of 11 fish. If each person caught at least one fish, then which of the following must be true?
   A. Somebody caught exactly 2 fish.
   B. Somebody caught exactly 3 fish.
   C. Somebody caught fewer than 3 fish.
   D. Somebody caught more than 3 fish.
   E. Two people each caught more than 1 fish.

26. The graph of \( y = Kx + N \) passes through \((-3, 20)\). What is the sum of \( K \) and \( N \) if the area of the triangle in the 1st Quadrant formed by the x-axis, y-axis, and the graph of \( y = Kx + N \) is 8 square units?
   A. 4
   B. 5
   C. 6
   D. 7
   E. 8

27. What is the sum of the solutions of \( \frac{2}{5}x - x^5 - 2 = 0 \)?
   A. 30
   B. 31
   C. -31
   D. 33
   E. -33
28. We roll two standard six-sided dice. One is red, one is green. What is the probability that the number on the top face of the red die is larger than the number on the top face of the green die?

A. \( \frac{1}{2} \)
B. \( \frac{5}{12} \)
C. \( \frac{4}{9} \)
D. \( \frac{13}{36} \)
E. Correct answer not given

29. What is the implied domain of \( y = \frac{\sqrt{x^2 - 16}}{x^2 + x} \)?

A. \( x \neq 0 \)
B. \(-4 < x < 4\)
C. \(-4 \leq x < -4 \) or \(-1 < x < 0 \) or \(0 < x \leq 4\)
D. \(-4 \leq x < 0 \) or \(0 < x \leq 4\)
E. Correct answer not given

30. What is the coefficient of the \( x^4 y^4 \) term in the expansion of \((2x - y)^9\)?

A. 4,032
B. -4,032
C. 2,016
D. -2,016
E. 48,384

31. Which of the following is equal to \( (\log_{25}(16))(\log_4(5))\)?

A. 1
B. \( \frac{1}{2} \)
C. 2
D. \( \frac{4}{5} \)
E. \( \frac{5}{4} \)
32. A, B, and C are the solutions of \( x^3 - 11x^2 + 36x - 36 = 0 \) where \( A < B < C \). What is \( 2A - B + 3C \)?
   A. 10  
   B. 13  
   C. -13  
   D. 19  
   E. -19

33. In triangle \( ABC \), \( m(\angle A) = 30^\circ \), \( a = 7.5 \) and \( \sin(C) = 0.6 \). Find \( c \).
   A. 8.2  
   B. 8.5  
   C. 8.8  
   D. 9.0  
   E. not enough information given to answer the question

34. A, B, C, D, E, F, G, H are evenly spaced points on the circle. What is \( \frac{m(\angle ADG)}{m(\angle ABF)} \) ?
   A. 1  
   B. \( \frac{1}{3} \)  
   C. \( \frac{2}{3} \)  
   D. \( \frac{1}{2} \)  
   E. Correct answer not given

35. What is the sum of all the real solutions of \( x^3 = \sqrt[3]{7x^3 + 8} \)?
   A. 7  
   B. -7  
   C. 9  
   D. -1  
   E. Correct answer not given

36. What is the sum of the solutions of the equation \( |3x - 5| = |15 - 2x| \)?
   A. 20  
   B. -14  
   C. 14  
   D. 6  
   E. -6
37. Find the x-coordinate of the point of intersection of \( y = \frac{x^2}{x^2 + 2x - 4} \) and its horizontal asymptote.

A. 1  
B. -1  
C. -2  
D. 2  
E. The graph does not cross its horizontal asymptote.

38. Find \( \left( \frac{1}{2} + \frac{\sqrt{3}}{2} \right)^{50} \).

A. \( \frac{1}{2} - \frac{\sqrt{3}}{2} \)  
B. \( \frac{1}{2} + \frac{\sqrt{3}}{2} \)  
C. 3000 + 6000i  
D. 3000 + 8660i  
E. Correct answer not given

39. Arrange \( 2^{600}, 3^{500}, 4^{400}, 5^{300} \) in order from smallest to largest.

A. \( 2^{600}, 3^{500}, 4^{400}, 5^{300} \)  
B. \( 3^{500}, 4^{400}, 5^{300}, 2^{600} \)  
C. \( 5^{300}, 4^{400}, 3^{500}, 2^{600} \)  
D. \( 2^{600}, 3^{500}, 4^{400}, 5^{300} \)  
E. Correct answer not given

40. Each of the following is the definition of a conic.

M: The collection of all points in the plane, the difference of whose distances to two fixed points is a constant.

N: The collection of all points in the plane, the sum of whose distances to two fixed points is a constant.

P: The collection of all points in the plane equidistant from a given point.

Q: The collection of all points in the plane that are equidistant from a given point and a given line.

Which of the following is the correct order of definitions for circle, parabola, ellipse, and hyperbola?

A. P, Q, M, N  
B. Q, P, M, N  
C. P, Q, N, M  
D. Q, P, N, M  
E. M, N, P, Q