INSTRUCTIONS: This is a 90-minute, 40-problem, multiple-choice examination. There are five (5) possible responses to each question or problem. You are to select the one (1) "best" answer to each. You may mark on the test booklet, and the back of each page may be used for additional work space. When you are sure of your answer, circle the letter of the choice you have made on the test booklet. After you have worked all problems you can work, transfer your answers to the score sheet which has your student number encoded. Darken completely the blank below the letter of your response to each question. Mark your answers boldly with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark below row 40. Your score will be computed by the formula 40 + (4C - 1), where C represents the number of correct answers and I represents the number of incorrect answers. If you can definitely rule out at least one choice it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. You may not use a calculator on this test.

Review and check your score sheet carefully. Your student identification number has been encoded on your red and white score sheet and has been checked by our marked-sense card reader. This number is in the "I.D. Number" section at the top; if you alter this number in any way you may disqualify yourself and your team from consideration for any awards.

When you complete your test, bring your answer sheet to the Test Monitor. You may keep your pencil and test booklet. You may leave the room after you have handed in your answer sheet.
1. Farmer Kipp decides to sell baked goods that his wife makes at his produce stand. He sells blueberry pies for $4.50 each, blackberry pies for $5.00 each, pecan pies for $6.00 each, and blueberry muffins for $4.00 for 6 muffins. On one of Farmer Kipp’s busiest days at the produce stand he sells 12 blueberry pies, 14 blackberry pies, 13 pecan pies, and 20 dozen muffins. How much did he receive for the baked goods?

A. $282  
B. $283  
C. $323  
D. $362  
E. $363

2. For which of the following values of x is this expression not real? \[ \frac{1}{\sqrt{\sqrt{7} - x}} \]

A. -2  
B. \(-\sqrt{7}\)  
C. 0  
D. 2  
E. 4

3. \(x = \frac{b}{a}\), \(a \neq b\) and \(a \neq 0\). Find \(\frac{a-b}{a+b}\).

A. \(\frac{x+1}{x-1}\)  
B. \(\frac{1-x}{1+x}\)  
C. \(\frac{1}{x}\)  
D. \(\frac{1}{x}\)  
E. \(\frac{1}{x}\)

4. What is the vertex of \(x^2 - 4x + 8y = -12\)?

A. \((-4,8)\)  
B. \((4,-8)\)  
C. \((-2,1)\)  
D. \((-2,-1)\)  
E. \((2,-1)\)
5. In an office, at various times during the day, the boss gives the secretary a letter to type, each time putting the letter on top of the pile in the secretary's in-box. When there is time, the secretary takes the top letter off the pile and types it. If there are five letters in all, and the boss delivers them in the order 1 2 3 4 5, which of the following could NOT be the order in which the secretary types them?

A. 1 2 3 4 5
B. 2 4 3 5 1
C. 3 2 4 1 5
D. 4 5 2 3 1
E. 5 4 3 2 1

6. Solve for x: \( \log(x) + \log(2x - 1) = \log(3) \).

A. \( -\frac{3}{2} \)
B. \(-1\)
C. \(1\)
D. \(\frac{3}{2}\)
E. \(\frac{4}{3}\)

7. If \( \tan(A) = \frac{2}{3} \), \( \pi \leq A \leq \frac{3\pi}{2} \), what is \( \sin(A) + \cos(A) \)?

A. \( \frac{5}{\sqrt{13}} \)
B. \( \frac{6}{\sqrt{13}} \)
C. \( -\frac{5}{\sqrt{13}} \)
D. \( -\frac{6}{\sqrt{13}} \)
E. \( -2 - \sqrt{5} \)

10. A triangle \( ABC \) with hypotenuse \( AB \) has side \( AC = 15 \). Altitude \( CH \) divides segments \( AH \) and \( HB \), with \( HB = 16 \). What is the area of triangle \( ABC \)?
8. Pegs are put into a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure, forming a quadrilateral. What is the area of this quadrilateral?

A. 4  
B. 4.5  
C. 5  
D. 5.5  
E. 6  

9. Function $f$ is defined by $f(x) = ax^2 - \sqrt{7}$ for some positive value of $a$. If $f(f(\sqrt{7})) = -\sqrt{7}$, find $a$.

A. $7 - \sqrt{7}$  
B. $\frac{1}{7}$  
C. $\frac{7 + \sqrt{7}}{7}$  
D. $\frac{\sqrt{7}}{7}$  
E. $\frac{7 - \sqrt{7}}{7}$

10. A right triangle ABC with hypotenuse AB has side AC = 15. Altitude CH divides AB into segments AH and HB, with HB = 16. What is the area of triangle ABC?

A. 120  
B. 144  
C. 150  
D. 216  
E. $144\sqrt{5}$
11. The following points are on circle C: (-1,8), (4,3), (2,7). Which one of the following points is on C?

A. \((-2, \sqrt{22})\)
B. \((-1 + \sqrt{22}, 5)\)
C. \((-1 - \sqrt{23}, 4)\)
D. \((0, 3 + \sqrt{23})\)
E. \((1, 3 - \sqrt{21})\)

12. When \(x^3 + Kx + 1\) is divided by \(x + 1\), the remainder is \(-4\). When \(x^3 - x^2 + Mx + 2\) is divided by \(x - 1\), the remainder is 1. Find \(K + M\).

A. \(-3\)
B. \(-2\)
C. 0
D. 1
E. 3

13. The first four terms of a geometric series are \(a\), \(5x\), \(bx\). What is the value of \(b/a\)?

A. 1.125
B. 1.25
C. 1.3
D. 1.45
E. 1.6

14. Find the distance between the vertices of the two parabolas represented by

\(y = 2x^2 - 8x - 1\) and \(y = \frac{1}{2}x^2 + 4x\).

A. \(\sqrt{193}\)
B. \(\sqrt{137}\)
C. \(\sqrt{118}\)
D. \(\sqrt{37}\)
E. \(\sqrt{17}\)
15. \( \cos(T) = \frac{1}{3}, \frac{3\pi}{2} \leq T \leq 2\pi \). In which quadrant is the angle 2T?

A. I  
B. II  
C. III  
D. IV  
E. Not enough information given

16. Which one of the following is an asymptote of \( \frac{(x + 2)^2}{9} - \frac{(y + 3)^2}{4} = 1 \)?

A. \(-9y + 4x = 36\)  
B. \(9y - 4x = 36\)  
C. \(-3y + 2x = -5\)  
D. \(3y - 2x = -5\)  
E. \(3y + 2x = -5\)

17. If \( 6(2x + \sqrt{7}) = R \), what is \( 3(6x + 3\sqrt{7}) \)?

A. \(\frac{R}{2}\)  
B. \(\frac{R}{3}\)  
C. \(\frac{3R}{2}\)  
D. \(\frac{2R}{3}\)  
E. \(\frac{3R}{4}\)

18. At the end of a professional bowling tournament, the top 5 bowlers have a play-off. First #5 bowls #4. The loser receives 5th prize and the winner bowls #3 in another game. The loser of this game receives 4th prize and the winner bowls #2. The loser of this game receives 3rd prize and the winner bowls #1. The winner of this game gets 1st prize and the loser gets 2nd prize. In how many orders can bowlers #1 through #5 receive the prizes?

A. 10  
B. 16  
C. 24  
D. 120  
E. None of the above.
19. Define the operation "#" by \( x \# y = 2x + y^2 - xy \) for all real numbers \( x \) and \( y \). For how many real numbers \( y \) does \( 3 \# y = 4 \)?

A. 0
B. 1
C. 2
D. 3
E. more than 3

20. What is the eccentricity of \( 9x^2 + 25y^2 + 36x - 150y = -361 \)?

A. \( \frac{2}{3} \)
B. \( \frac{3}{4} \)
C. \( \frac{4}{5} \)
D. \( \frac{5}{6} \)
E. \( \frac{6}{7} \)

21. Betty Smart and her parents are vacationing at the beach. On Monday, Mr. and Mrs. Smart drive to the beach in 3.5 hours. On Tuesday, Betty makes the same trip in 3 hours. Betty drives an average of 10 mph faster than her parents. How far is the beach?

A. 60 miles
B. 120 miles
C. 165 miles
D. 180 miles
E. 210 miles

22. \( a \) is the largest solution and \( b \) is the smallest solution for the equation \( x + 5 = \frac{2}{x} \). What is \( 6a - 2b \)?

A. \( -5 + 8\sqrt{3} \)
B. \( -5 + 5\sqrt{3} \)
C. \( -10 + 4\sqrt{3} \)
D. \( -10 + 2\sqrt{3} \)
E. \( -10 + 2\sqrt{17} \)
23. If \( \sin(x) = \frac{2}{3}, \quad \frac{\pi}{2} \leq x \leq \pi, \) and \( \cos(y) = \frac{1}{3}, \quad \frac{3\pi}{2} \leq y \leq 2\pi, \) what is \( \sin(x + y) \)?

A. \( -\frac{2+2\sqrt{10}}{9} \)
B. \( -\frac{2+2\sqrt{10}}{9} \)
C. \( \frac{2-2\sqrt{10}}{9} \)
D. \( \frac{2+2\sqrt{10}}{9} \)
E. \( \frac{4\sqrt{10}}{9} \)

24. Four positive integers, when added 3 at a time, give sums of 124, 132, 169, and 187. What is the smallest of the four positive integers?

A. 17
B. 27
C. 37
D. 47
E. 57

25. \( g(x) = -x^2 + 14x - 1 \) lies above \( f(x) = x^4 - 4x^3 + 3x^2 - 2x - 1 \) only on \( a < x < b \). What is \( 2a + 3b? \)

A. 6
B. 8
C. 10
D. 12
E. 14

26. In the figure, the sum of the distances AD and BD is

A. between 10 and 11.
B. 12.
C. between 15 and 16.
D. between 16 and 17.
E. 17.
27. What is the range of \( f(x) = \frac{2x - 7}{x + 4} \)?

A. all real numbers except 2
B. all real numbers except negative 2
C. all real numbers except 4
D. all real numbers except negative 4
E. all real numbers except 0

28. What is the sum of the solutions of \( 2\sin^3(x) + \sin^2(x) - \sin(x) = 0 \), where \( 0 \leq x < 2\pi \)?

A. \( \frac{7\pi}{2} \)
B. \( \frac{10\pi}{3} \)
C. \( \frac{19\pi}{6} \)
D. \( \frac{21\pi}{3} \)
E. \( \frac{23\pi}{6} \)

29. Simplify \( \sqrt{\frac{x}{9} + \frac{x}{16}} \).

A. \( \frac{\sqrt{2x}}{5} \)
B. \( \frac{5\sqrt{x}}{12} \)
C. \( \frac{7\sqrt{x}}{12} \)
D. \( \frac{5x}{7} \)
E. \( \frac{|x|}{12} \)
30. Let S be the statement

"If the sum of the digits of the whole number \( n \) is divisible by 6, then \( n \) is divisible by 6."

Which of the following is a value of \( n \) which shows S to be false?

A. 30  
B. 33  
C. 40  
D. 42  
E. None of the above.

31. There are two natural ways to inscribe a square in a given isosceles right triangle. If it is done as in Figure 1 below, then the area of the square is 18 square centimeters. What is the area (in square centimeters) of the square inscribed in the same triangle ABC as shown in Figure 2 below?

A. 12\(\sqrt{2}\)  
B. 16  
C. 18  
D. 20 - 2\(\sqrt{2}\)  
E. None of the above.

32. For \( i = \sqrt{-1} \), which expression is equivalent to \( \frac{x^2 - 2x + 2}{x - (1 + i)} \)?

A. \( x - 1 - i \)  
B. \( x - 1 + i \)  
C. \( 3x - 2i \)  
D. \( 3x + 2i \)  
E. \( i \)
33. Which one of the following functions has exactly one complete period on 
\[ \frac{\pi}{2} \leq x \leq \frac{9\pi}{2} \]?

- A. \( y = \sin \left( 4x + \frac{\pi}{2} \right) - 3 \)
- B. \( y = -3\sin \left( 4x - \frac{\pi}{4} \right) + 2 \)
- C. \( y = 3\sin \left( \frac{x}{4} + \frac{\pi}{2} \right) - 1 \)
- D. \( y = -5\sin \left( 2x - \frac{\pi}{4} \right) + 2 \)
- E. \( y = 5\sin \left( \frac{x}{2} + \frac{\pi}{4} \right) - 3 \)

34. If \( a > 0, b > 0, \) and \( \log_3 \left( \frac{1}{9} \right) + x = 3, \) find \( \frac{a^{a-b}}{b^{a-2}}. \)

- A. \( ab^6 \)
- B. \( ab^{100} \)
- C. \( a^3b^{19} \)
- D. \( a^7b^6 \)
- E. \( a^7b^{100} \)

35. In the triangle ABC shown, D is some interior point, and \( x, y, z, w, \) are the measures of angles in degrees. Solve for \( x \) in terms of \( y, z, \) and \( w. \)

- A. \( w - y - z \)
- B. \( w - 2y - 2z \)
- C. \( 180 - w - y - z \)
- D. \( 2w - y - z \)
- E. \( 180 - w + y + z \)
36. Which expression is equivalent to: \( \frac{x^3 - 8}{x^2 - 2x - 8} + \frac{x^2 + 2x + 4}{x + 2} - \frac{x - 4}{x - 2} \)?

A. \( \frac{12}{x^2 - 6x + 8} \)
B. \( \frac{4x - 12}{x^2 - 6x + 8} \)
C. \( \frac{20 - 12x}{x^2 - 6x + 8} \)
D. \( \frac{-3x}{2} \)
E. \( \frac{4}{x - 2} \)

37. A ball thrown horizontally from the top of a 100 ft. cliff follows a parabolic curve with vertex at the top of the cliff and axis along the side of the cliff. The ball passes through a point 10 ft. from the face of the cliff when it is a vertical distance of 25 ft. from the top. How far from the base of the cliff will the ball land?

A. 10 ft.
B. 15 ft.
C. 20 ft.
D. 25 ft.
E. 30 ft.

38. At 1:00 p.m. plane A flies over point M flying due east at 100 mph. 30 minutes later, plane B flies over point M flying northwest at 60 mph. Assuming that the planes maintain their speed and direction, how far apart are the two planes at 3:00 p.m. on the same day?

A. \( \sqrt{48,100 + 18,000\sqrt{2}} \)
B. \( \sqrt{48,100 - 18,000\sqrt{2}} \)
C. \( \sqrt{66,100\sqrt{2}} \)
D. \( \sqrt{30,100\sqrt{2}} \)
E. \( \sqrt{66,100 + \sqrt{2}} \)
39. Which of the following is the least integer \( n \) for which \( n^{100} > 11^{200} \)?

A. 3  
B. 5  
C. 7  
D. 9  
E. 11  

40. AC is a diameter of the circle shown in the diagram, and B is another point on the circle. If the length of AC is 5 and the length of AB is 1, what is the length of BC?

A. \( 6 - \sqrt{2} \)  
B. \( 2\sqrt{6} \)  
C. \( 2\sqrt{5} \)  
D. \( 3\sqrt{2} \)  
E. None of the above.