Georgia Southwestern State University
Mathematics Tournament
Test Booklet
2009

INSTRUCTIONS: This is a 90-minute, 40-problem, multiple choices exam. There are five (5) possible responses to each question. You are to select the one best answer to each question. You may mark on the test booklet and use the back of each page for additional work. When you are sure of your answer, circle the letter of the choice you have made in the test booklet. After you have worked all problems you can work, transfer your answer to the score sheet which has your student number encoded. Darken completely the blank below the letter of your response to each question with a No. 2 pencil provided by GSW. If you decide to change your answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark below row 40. Your score will be computed by the formula $40 + (4C - I)$, where C is the number of correct answers and I is the number of incorrect answers. If you can definitely rule out at least one choice it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. You may not use a calculator on this test.

Review and check your score sheet carefully. Your student identification number has been encoded on your red and white score sheet and has been checked by our marked sense card reader. This number is in the "I.D. Number" section at the top; if you alter this number in any way, you may **disqualify yourself and your team from consideration for any awards.**

When you complete your test, bring your answer sheet to the Test Monitor. You may keep your pencil and test booklet. You may leave the exam room after you have handed in your answer sheet.

1. Among other things, farmer Kipp sells corn, peas, and tomatoes. He sells corn for \$3.90/dozen, peas for \$3.20/lb, and tomatoes for \$1.20/lb. How much does it cost to buy 18 ears of corn, 4 lbs of peas, and 3 lbs of tomatoes from Farmer Kipp?

   (a) \$21.25 (b) \$21.80 (c) \$22.25 (d) \$22.80 (e) \$23.25

2. If you drive a race car 20 miles at 40 mph, then another 20 miles at 60 mph and finally 20 more miles at 120 mph; what is your average speed for the entire 60 mile trip in mph?

   (a) 30 (b) 40 (c) 50 (d) 60 (e) 70

3. What is $\dfrac{x^3-8}{3x^2+6x+12} \div \dfrac{2x^2-8}{x^2+5x+6}$ when expressed as a fraction in lowest terms?

   (a) $\dfrac{x+3}{6}$ (b) $\dfrac{x^2-x-6}{3x+6}$

   (c) $\dfrac{x^2+x-6}{3x-6}$ (d) $\dfrac{6}{x+3}$

   (e) correct answer not given

4. What is the sum of the solutions of $|x^2 + 4x - 5| = |x^2 + 2x + 9|$?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

5. What is the sum of the linear factors of $x^3 + x^2 - 9x - 9$?

(a) $3x + 1$ (b) $3x - 1$ (c) $3x + 7$ (d) $3x - 7$
(e) $3x + 8$

6. The largest solution of the equation $8x^2 + 16x - 120 = 0$ is in which of the following intervals?

(a) [-5,-3] (b) (-3,-1]
(c) (-1,3) (d) [3,7]
(e) [7,11]

7. What is the solution of the inequality $x^2 + 5x + 6 \geq 0$?

   (a) $(-\infty, 2] \cup [2, \infty)$ (b) $(-\infty, -3] \cup [1, \infty)$

   (c) $[-3, -2]$ (d) $[2, 3]$

   (e) $(-\infty, -3] \cup [-2, \infty)$

8. A jar contains 20 coins: some pennies, some nickels. Each of the following could be the ratio of the number of pennies to number of nickels EXCEPT:

   (a) 1:1 (b) 1:3 (c) 4:1 (d) 5:1 (e) 9:1

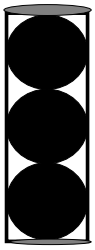
9. Which of the following is in the domain of $\dfrac{\sqrt{x-2}}{x-7}$?

   (a) 7 (b) 2 (c) 1.5 (d) 1 (e) -7

10. How many integers from 1 to 999 are divisible by 40 but not by 30 ?

    (a) 24 (b) 22 (c) 18 (d) 16 (e) 14

11. Three tennis balls fit snugly inside a right circular cylinder as shown in the figure. What fraction of the total interior volume do the balls occupy?



(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$ (e) $\frac{5}{6}$

12. If $x < -2|y|$, which of the following statements is always true?

(a) $-x < y < 0$ (b) $-x > y > 0$

(c) $-x > 0$ (d) $-2y < x$

(e) $y > 0$

13. If $f(x+3) = g(3-x)$, then what is $g(x)$?

(a) $f(-x+6)$ (b) $f(x-3)$

(c) $f(-x-6)$ (d) $f(x+3)$

(e) $f(x+6)$

14. The area enclosed by $y = x + 2, y = -x + 2$ and $y = b$ is 16 square units. Which of the following could be the value of b?

(a) 7 (b) 5 (c) 3 (d) -1 (e) -2

15. What is the length of the longest cord that can be stretched into a straight line segment from a corner on the floor to an opposite corner on the ceiling in a $20 \times 20 \times 10$ rectangular-prism-shaped room?

(a) 30 (b) $\frac{50}{\sqrt{2}}$ (c) $10\sqrt{2}$ (d) $20\sqrt{2}$ (e) 25

16. Let x and y be two positive integers less than 11 and $3x + 7y$ is a multiple of 11. Which of the following must be a multiple of 11?

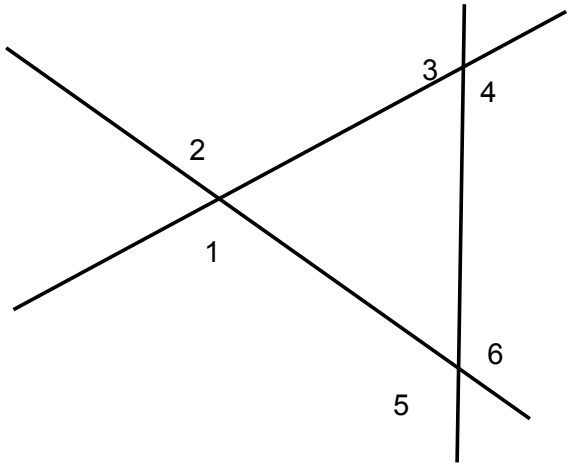
(a) $3x - 4y$ (b) $3x + 4y$ (c) $2x - 4y$ (d) $2x + 4y$

(e) Correct answer not given

17. If $F(1) = 3$ and $F(n) = F(n-1) + \frac{1}{2}$, what is the value of $F(101)$?

(a) 50 (b) 51 (c) 52 (d) 53 (e) 54

18. Three lines intersect at three points as shown in the figure forming 12 angles. What is the sum of the measures of the angles labeled 1 through 6?



(a) 540^o (b) 720^o (c) 840^o (d) 900^o

(e) Cannot be determined from the information given

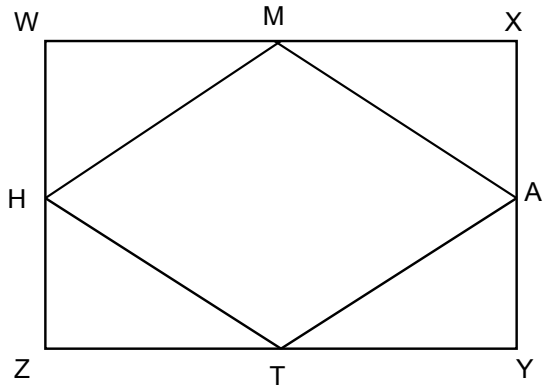
19. A drawer contains 3 black, 2 blue and 4 red socks. If two socks are selected randomly from the drawer, what is the probability that they are of the same color?

(a) $\frac{5}{18}$ (b) $\frac{2}{9}$ (c) $\frac{1}{1}$ (d) $\frac{2}{72}$ (e) $\frac{20}{81}$

20. Which of the following **cannot** be a root of a polynomial in x of the form $6x^{10} + ax^6 + b$, where a and b are integers ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) -6 (e) 1

21. WXYZ is a rectangle and M,A,T and H are the midpoints of the four sides. The perimeter of WXYZ is 28 and the ratio of its length to its width is $4 : 3$. What is the perimeter of the quadrilateral MATH?



(a) 28 (b) 25 (c) 22 (d) 20 (e) 16

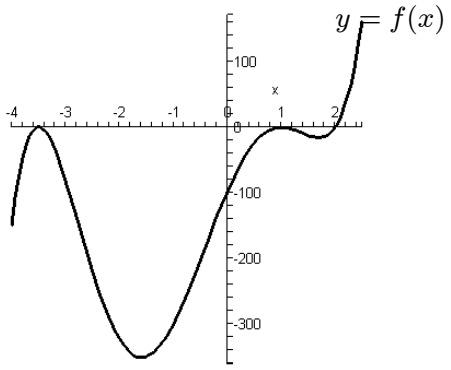
22. If the product ab is negative, which of the following **cannot** be negative?

(a) $a - b$ (b) $a^2 - b^2$

(c) $a^2 - b^2 + 2ab$ (d) $a^2 + b^2 - 2ab$

(e) $b - a$

23. Following is the graph of a polynomial function $y = f(x)$ of degree n. Which of the following values is a possible value for n?



(a) 3 (b) 4 (c) 5 (d) 6 (e) 8

24. What is the value of $(3003)^3 - (3004)(3003)(3002)$?

(a) 0 (b) 3000 (c) 3002 (d) 3003 (e) 3004

25. The length of each side of an equilateral triangular field is 2 miles. A deer starts walking from a point outside the field around the field, maintaining a distance 2 miles from the field at all times. How far does the deer walk in miles when it comes back to its original position?

(a) 12 (b) $12 + 2\pi$ (c) 6 (d) $6 + 4\pi$ (e) 9

26. The circle $x^2 + y^2 + 4x - 6y = 12$ has a center (a, b) and radius r. What is $a + b + r$?

(a) 4 (b) 6 (c) 8 (d) 10 (e) 12

27. A parabola with a maximum point passes through the points $(0, 5), (1, 8)$ and $(-2, -7)$, what is the $y-$value of the point on the graph of the parabola when the x value is 2?

(a) 3 (b) 5 (c) 7 (d) 9 (e) 11

28. How far apart are the foci of $\dfrac{(x+3)^2}{9} - \dfrac{(y-2)^2}{16} = 1$

(a) 2 (b) 6 (c) 8 (d) 10 (e) 12

29. The vertices of an ellipse on the major axis are $(6, -2)$ and $(-4, -2)$ and on the minor axis are $(1, 2)$ and $(1, -6)$. What is the smallest value of y on the graph of the ellipse when $x = 2$?

(a) $\dfrac{2 - 4\sqrt{6}}{5}$ (b) $\dfrac{2 - 6\sqrt{6}}{5}$ (c) $\dfrac{6 - 4\sqrt{6}}{5}$ (d) $\dfrac{8 - 10\sqrt{6}}{5}$ (e) $\dfrac{-10 - 8\sqrt{6}}{5}$

30. There are three bags: one with 20 red marbles, one with 20 green marbles and one with 10 of each color. Each is marked "Red marbles", "Green marbles" or " Mixed marbles" and none of the labels are correct. What is the fewest number of marbles that can be drawn in order to determine the correct labels?

(a) 1 (b) 2 (c) 3 (d) 16 (e) 18

31. What is the value of
$(\tan 20^o)(\tan 30^o)(\tan 40^o)(\tan 50^o)(\tan 60^o)(\tan 70^o)$?

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$ (e) $\frac{1}{2}$

32. How many equilateral triangles can be formed using the vertices of a cube?

(a) 2 (b) 4 (c) 6 (d) 8 (e) 12

33. How many values of x in $[-2\pi, 2\pi]$ will satisfy the equation $\cos^2 x + 2\sin^2 x = 1$?

(a) 1 (b) 3 (c) 5 (d) 7

(e) Infinitely many

34. The y-intercept of the graph of $y = 2x + b$ is 15 units below the y-intercept of the graph of $y = qx + t$. If the two graphs intersect at $(3, -4)$, what is q?

(a) -2 (b) -3 (c) -4 (d) -5 (e) -6

35. A tree contains a certain number of bananas. Monkey Able eats one-third of the bananas; Monkey Baker then eats one-fourth of the remaining bananas and finally Monkey Charlie eats one-half of the remaining bananas. If there are now 6 bananas left on the tree, how many were there at the start?

(a) 72 (b) 64 (c) 42 (d) 36 (e) 24

36. If $\cot\theta = -\dfrac{10}{3}$, where $\dfrac{\pi}{2} \leq \theta \leq \pi$, then $\sin\theta$ is

(a) $\dfrac{3}{\sqrt{109}}$ (b) $\dfrac{\sqrt{109}}{3}$ (c) $-\dfrac{3}{\sqrt{109}}$ (d) $-\dfrac{\sqrt{109}}{3}$ (e) $\dfrac{3}{\sqrt{10}}$

37. How many points with integer coordinates lie either on or in the interior of the circle $x^2 + y^2 = 25$?

(a) 55 (b) 57 (c) 60 (d) 65 (e) 81

38. All the following statements are always true for an angle A, where $0 < A < \frac{\pi}{2}$, EXCEPT:

(a) $\cos^2 A = 1 - \sin^2 A$

(b) $\sin 2A = 2 \sin A \cos A$

(c) $\tan^2 A + 1 = \sec^2 A$

(d) $\cos^2 A - \sin^2 A = 1$

(e) $\csc^2 A - 1 = \cot^2 A$

39. What is the mean of the set consisting of the least 10 prime numbers?

(a) 10 (b) 10.1 (c) 10.5 (d) 11.5 (e) 12.9

40. What is the period of the function $y = 2 \sin (2\pi x + 3)$?

(a) π (b) 1 (c) 2π (d) 2 (e) $\frac{\pi}{2}$