

Georgia Southwestern State University
Mathematics Tournament
Test Booklet
2010

INSTRUCTIONS: This is a 90-minute, 40-problem, multiple-choice exam. There are five (5) possible responses to each question. You are to select the one best answer to each question. You may mark on the test booklet and use the back of each page for additional work. When you are sure of your answer, circle the letter of the choice you have made in the test booklet. After you have worked all problems you can work, transfer your answers to the score sheet which has your student number encoded. Darken completely the blank below the letter of your response to each question with a No. 2 pencil provided by GSW. If you decide to change your answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark below row 40. Your score will be computed by the formula $40 + (4C - I)$, where C is the number of correct answers and I is the number of incorrect answers. If you can definitely rule out at least one choice it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. You may not use a calculator on this test.

Review and check your score sheet carefully. Your student identification number has been encoded on your red and white score sheet and has been checked by our marked sense card reader. This number is in the “I.D. Number” section at the top; if you alter this number in any way, you may **disqualify yourself and your team from consideration for any awards.**

When you complete your test, bring your answer sheet to the Test Monitor. You may keep your pencil and test booklet. You may leave the exam room after you have handed in your answer sheet.

1. Farmer Cliett sold 5 bushels of peas, 4 bushels of peaches and 2 bushels of blueberries to a distributor. If he sold the peas for \$25 per bushel, the peaches for \$24 per bushel, and the blueberries for \$75 per bushel, how much did Farmer Cliett receive from this transaction?
(a) \$371 (b) \$375 (c) \$380 (d) \$381 (e) \$385

2. The average of 8 numbers is 7. If two of these eight numbers are removed, then the average of the remaining six numbers is 6, what is the sum of the removed numbers?
(a) 14 (b) 15 (c) 17 (d) 20 (e) 22

3. Which is greater?
(a) The radius of a circle of circumference 2π
(b) The length of a diagonal of a unit square
(c) The height of an equilateral triangle with sides of length 2
(d) The length of a diagonal of a 0.9×1.2 rectangle
(e) a, b, c and d are equal

4. A number is palindromic if it reads the same forwards and backwards. Three examples of six-digit palindromic numbers are 123321, 221122, and 555555. How many six-digit numbers are palindromic?
(a) 1,000,000 (b) 900,000 (c) 9,000 (d) 900 (e) 90

5. If $f(x) = ax^2 + bx + c$, $f(-1) = 1$, $f(1) = -5$ and $f(2) = -2$, what is $2a + b - 3c$?
(a) 9 (b) 10 (c) 11 (d) 12 (e) 13

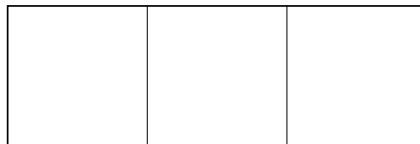
6. $\frac{\tan\left(\frac{5\pi}{4}\right) + \left(2 \sec\left(\frac{\pi}{3}\right)\right)^{\sin\left(\frac{\pi}{6}\right)}}{\cos\left(-\frac{11\pi}{2}\right) \cot\left(\frac{12\pi}{7}\right) + \sin\left(-\frac{4\pi}{3}\right)}$ equals which of one of the following?

- (a) $4\sqrt{2}$ (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $2\sqrt{2}$ (e) $4\sqrt{3}$

7. What is the distance from the midpoint of the line segment whose end points are $(-5, 10)$ and $(7, -4)$ to the origin?

- (a) $2\sqrt{2}$ (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$ (e) $2\sqrt{3}$

8. Three adjacent unit squares (1×1), aligned in a straight row, have a perimeter of 8. What is the perimeter of 100 unit squares, so aligned?



- (a) 400 (b) 302 (c) 300 (d) 202 (e) 200

9. $\frac{6x^2 - 19x + 15}{8x^2 - 18} \div \frac{4x^3 - 6x^2 + 9x}{16x^3 + 54}$ equals which one of the following?

(a) $\frac{2x - 3}{3x - 5}$ (b) $\frac{2x - 3}{3x}$

(c) $\frac{3x + 5}{2x + 3}$ (d) $\frac{3x - 5}{x}$

(e) correct answer not given

10. The perimeter of a rectangle is 24 inches and the measure of each side is a positive integer. How many such rectangles with different areas are possible?

- (a) 2 (b) 4 (c) 6 (d) 8 (e) 12

11. What is the coefficient of x^3y^4 in the simplified expansion for $(x - y)^7$?

- (a) 35 (b) 21 (c) -20 (d) -21 (e) -35

12. What is the sum of the solutions of the equation:

$$x^3 + 13x^2 + 39x + 27 = 0?$$

- (a) -10 (b) -11 (c) -12 (d) -13 (e) -14

13. If $f(x) = 2x^2 - 6x + 4$, what is $\frac{f(x+h) - f(x)}{h}$?

- (a) 1 (b) $4x - 2h - 6$
(c) $4x + 2h - 6$ (d) $4x + 2h + 6$
(e) $4x - 2h + 6$

14. A radiator contains 8 quarts of a mixture of water and antifreeze. If 40% of the mixture is antifreeze, how much of the mixture should be drained and replaced by pure antifreeze in order that the resultant mixture will contain 60% antifreeze?

- (a) $\frac{5}{3}$ quarts (b) 2 quarts (c) $\frac{7}{3}$ quarts (d) $\frac{8}{3}$ quarts (e) 3 quarts

15. If $\tan(T) = -\frac{1}{3}$, $\frac{\pi}{2} \leq T \leq \pi$, what is $\sin(2T) + \cos(2T)$?

- (a) $-\frac{1}{5}$ (b) $\frac{1}{5}$ (c) $-\frac{2}{5}$ (d) $\frac{2}{5}$ (e) $\frac{3}{5}$

16. A projectile is fired horizontally at a target and the sound of its impact is heard 1.5 seconds later. If the speed of the projectile is 3300 ft/sec, and that of sound is 1100 ft/sec, how far is the target?

- (a) 1,237 ft (b) 1,237.5 ft (c) 1,238 ft (d) 1,238.5 ft (e) 1,239 ft

17. Which one of the following is an asymptote of

$$9x^2 - 4y^2 - 18x - 8y = 31?$$

- (a) $2y - 3x = -5$ (b) $2y + 3x = 5$
(c) $2y + 3x = -1$ (d) $2y - 3x = 1$
(e) $3y + 2x = 5$

18. What is the equation of the circle centered at the point (2,0) that is tangent to the line $4y + 3x = 31$?

- (a) $x^2 - 4x + y^2 = 18$ (b) $x^2 - 4x + y^2 = 19$
(c) $x^2 - 4x + y^2 = 20$ (d) $x^2 - 4x + y^2 = 21$
(e) $x^2 - 4x + y^2 = 22$

19. If $f(x) = mx + b$ is the equation of the line passing through both the y -intercept and the x -intercept with the largest value of x of $g(x) = x^3 + x^2 - 4x - 4$, what is $f(3)$?

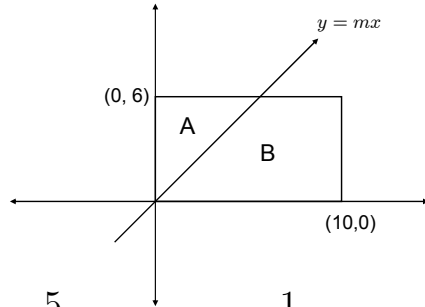
- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

20. What is the sum of the solutions of the following equation for $0 \leq T < 2\pi$?

$$\sin^2(T) + \cos(T) + \cos^2(T) = \frac{1}{2}$$

- (a) π (b) $\frac{7\pi}{6}$ (c) 2π (d) $\frac{7\pi}{3}$ (e) $\frac{11\pi}{6}$
21. If $f(x)$ and $g(x)$ are two different polynomials such that $f(x) \neq g(x)$ for at least two values of x , and each of these polynomials has two distinct real zeroes. What are the possible numbers of zeroes of the polynomial obtained by multiplying $f(x)$ and $g(x)$, that is the polynomial $f(x)g(x)$?
- (a) 2, 3 and 4 (b) 3 and 4 only (c) 2 and 3 only (d) 3 only
(e) 4 only
22. Two five digit numbers, both multiples of 9 are given as $M2348$ and $2344N$, where M and N are positive integers. What is $M + N$?
- (a) 4 (b) 5 (c) 6 (d) 7 (e) 8
23. If A is the farthest point on the circle $x^2 - 6x + y^2 + 4y - 12 = 0$ from the point $B(-2, -2)$, what is the distance from the point B to A ?
- (a) 8 (b) 10 (c) 12 (d) 14 (e) 16
24. A drawer has 14 red, 12 white, and 10 blue socks. If Mr. XYZ were blindfolded so that he can not tell the color of the socks pulled from the drawer, what is the least number of socks that Mr. XYZ needs to pull out from the drawer to be certain of having a pair of any color?
- (a) 4 (b) 25 (c) 27 (d) 32 (e) 36

25. The line $y = mx$ passes through a rectangle with vertices $(0, 0)$, $(10, 0)$, $(10, 6)$ and $(0, 6)$ as shown in the figure. If the line cuts the rectangle into two parts A and B , find the value of m so that the ratio of area of A and B will be 1:3.



- (a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$ (e) $\frac{1}{6}$

26. John's dad is twice as old as John. Eight years ago, John was twice as old as his sister Emma. In 10 years, John's dad will be twice as old as Emma. How old is John's dad now?

- (a) 32 (b) 36 (c) 40 (d) 44 (e) 48

27. Two poles, 40 feet tall and 10 feet tall are 40 feet apart. Support cables are placed from the top of one pole to the bottom of the other pole. How high above the ground is the intersection of the cables?

- (a) 4 feet (b) 5 feet (c) 6 feet (d) 7 feet (e) 8 feet

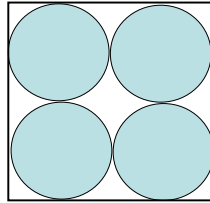
28. Which is greater?

- (a) $\tan 30^\circ$ (b) $\sin 30^\circ$ (c) $\cos 30^\circ$ (d) $\csc 30^\circ$ (e) $\sec 30^\circ$

29. $1 + 2 + 3 + 4 + \dots + 397 + 398 + 399 + 400 = ?$

- (a) 80,200 (b) 78,000 (c) 60,300 (d) 50,500 (e) 40,700

30. Four congruent circular cookies are fitted snugly into a flat square box so that they tangentially touch each other and the sides of the box. What part of the bottom of the box is covered?

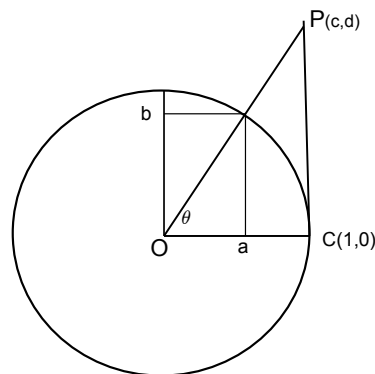


- (a) $\frac{\pi}{8}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{3}{4}$ (e) $\frac{7}{8}$

31. Which value for the area is found using the formula “the height multiplied by the average of the length of the bases”?

- (a) Square only (b) Rectangle only
 (c) Parallelogram only (d) Trapezoid only
 (e) All of above

32. Shown are a unit circle (of radius one) and right angle OCP . O is the circle’s center. What is the value of $\tan(\theta)$?



- (a) a (b) b (c) c (d) d (e) a/b

33. What is the measure of the largest angle in a 7-8-9 scalene triangle?

(a) $\cos^{-1}\left(\frac{8}{9}\right)$ (b) $\cos^{-1}\left(\frac{64}{81}\right)$ (c) $\cos^{-1}\left(\frac{81}{49}\right)$ (d) $\cos^{-1}\left(\frac{32}{112}\right)$

(e) $\cos^{-1}\left(\frac{64}{144}\right)$

34. What is the length of the longest side of a $44^\circ - 66^\circ - 70^\circ$ scalene triangle with a shortest side of 12?

(a) $\frac{12 \sin 44^\circ}{\sin 70^\circ}$ (b) $\frac{12 \sin 70^\circ}{\sin 44^\circ}$ (c) $\frac{12 \sin 66^\circ}{\sin 44^\circ}$ (d) $\frac{12 \sin 44^\circ}{\sin 66^\circ}$

(e) $12 \sin 44^\circ \sin 70^\circ$

35. $300,000,001 \times 299,999,999 = ?$

(a) $8.1 \times 10^{16} - 1$

(b) $3.1 \times 10^{17} + 1$

(c) $9.1 \times 10^{16} - 1$

(d) $3.0 \times 10^{17} + 1$

(e) $9.0 \times 10^{16} - 1$

36. How many integers between 100 and 2000 are multiples of 12?

(a) 166

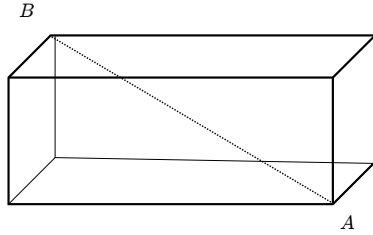
(b) 162

(c) 160

(d) 158

(e) 154

37. In the given rectangular prism, the area of the faces are 18, 90, 20 respectively. What is the length of the diagonal AB ?



- (a) 64 (b) 128 (c) $\sqrt{128}$ (d) 185 (e) $\sqrt{185}$
38. How many points of intersection are shared by the graphs of $x^2 - y = 4$ and $|x| + |y| = 4$?
- (a) 0 (b) 2 (c) 3 (d) 4 (e) 5
39. The sum of two positive numbers is 9 and their product is 15. What is their difference?
- (a) $\sqrt{3}$ (b) 3 (c) $\sqrt{21}$ (d) 6 (e) 21
40. For which value of c will the quadratic equation $3x^2 + 2x - c = 0$ have NO real solutions?
- (a) $c \leq \frac{1}{3}$ (b) $c < \frac{1}{3}$ (c) $c \leq -\frac{1}{3}$ (d) $c < -\frac{1}{3}$ (e) $c \leq 0$