Georgia Southwestern State University
Mathematics Tournament
Test Booklet
2011

INSTRUCTIONS: This is a 90-minute, 40-problem, multiple-choice exam. There are five (5) possible responses to each question. You are to select the one best answer to each question. You may mark on the test booklet and use the back of each page for additional work. When you are sure of your answer, circle the letter of the choice you have made in the test booklet. After you have worked all problems you can work, transfer your answers to the score sheet which has your student number encoded. Darken completely the blank below the letter of your response to each question with a No. 2 pencil provided by GSW. If you decide to change your answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark below row 40. Your score will be computed by the formula $40 + (4C - I)$, where C is the number of correct answers and I is the number of incorrect answers. If you can definitely rule out at least one choice it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. You may not use a calculator on this test. Figures are not scaled.

Review and check your score sheet carefully. Your student identification number has been encoded on your red and white score sheet and has been checked by our marked sense card reader. This number is in the "I.D. Number" section at the top; if you alter this number in any way, you may **disqualify yourself and your team from consideration for any awards**.

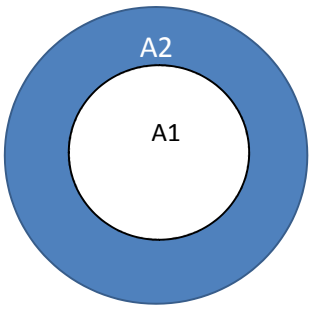
When you complete your test, bring your answer sheet to the Test Monitor. You may keep your pencil and test booklet. You may leave the exam room after you have handed in your answer sheet.

PLEASE DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

1. Farmer Gugg decides to sell some livestock to help pay for increased fuel costs. He sells 6 heads of cattle, 7 hogs, and 25 chickens. How much does he receive if he gets \$900 for each cow, \$250 for each hog, and \$6 for each chicken?

   (a) \$7300 (b) \$7000 (c) \$6800 (d) \$6500 (e) \$6300

2. Two concentric circles have radii that differ by 2 units. What is the smaller radius if the area of the region in common to the two circles is the same as the area of the larger circle less the area of the interior in the smaller circle? That is, $A1 = A2$.



   (a) $\sqrt{2}$ (b) 2 (c) $\frac{7}{2}$ (d) 4 (e) $2+2\sqrt{2}$

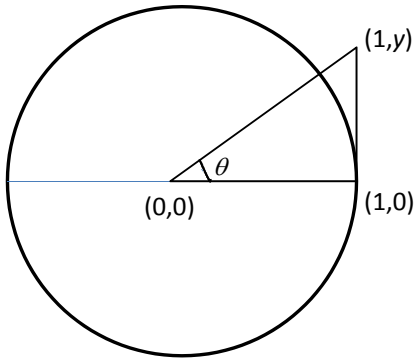
3. What is the length of the longest, straight thin metal rod that will fit in a rectangular room that has a 12ft $\times$20ft level floor and a flat 9 ft high ceiling?

   (a) 15 ft (b) $\sqrt{544}$ ft (c) $\sqrt{600}$ ft (d) 24 ft (e) 25 ft

4. $\dfrac{\sin^2\theta + \tan^2\theta + \cos^2\theta}{\sec^2\theta} = ?$

   (a) $\cos\theta$ (b) $\sin\theta$ (c) $\tan\theta$ (d) 1 (e) 2

5. Given: a unit circle and a right triangle. Find the value of y.



(a) 1 (b) $\frac{1}{2}$ (c) $\sin\theta$ (d) $\cos\theta$ (e) $\tan\theta$

6. Find all the solutions to the equation:$(2x-3)^{(x^2-9)} = 1$. Next, multiply them together to form a product. What is this product?

(a) -1 (b) -4 (c) -6 (d) -9 (e) -18

7. Al has twice as much money as Bea. Bea has three times as much money as Cy. Cy has \$10 more than Dee. Together they have \$441. How much money does Bea have?

(a) \$100 (b) \$123 (c) \$128 (d) \$135 (e) \$150

8. What is the distance from the midpoint of the line segment whose end points are $(-5, 10)$ and $(7, -4)$ to the origin?

(a) $4\sqrt{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{10}$ (d) $3\sqrt{3}$ (e) $2\sqrt{2}$

9. Permutations are the arrangements of n distinct objects taken r at a time when the drawing is done with respect to order and without replacement. The number of these is sometimes symbolized, $P(n, r)$. Combinations are the arrangements of n distinct objects taken r at a time when the drawing is done without respect to order and without replacement. The number of these is sometimes symbolized, $C(n, r)$. Evaluate: $\dfrac{P(12, 4)}{C(10, 3)}$

(a) 99 (b) 72 (c) 55 (d) 33 (e) 12

10. Using 3 evenly-balanced, regular, 6-faced game dice, what is the probability of rolling a "seven"? That is the total number of spots on the upper faces of the 3 dice is 7.

(a) $\dfrac{1}{6}$ (b) $\dfrac{5}{216}$ (c) $\dfrac{5}{72}$ (d) $\dfrac{7}{216}$ (e) $\dfrac{2}{9}$

11. Eccentricity of a conic is defined to be $e = \dfrac{c}{a}$. What is the eccentricity of the ellipse: $\dfrac{(x-3)^2}{4} + \dfrac{(y-5)^2}{9} = 1$?

(a) 5 (b) $\dfrac{\sqrt{5}}{3}$ (c) $\dfrac{9}{4}$ (d) $\dfrac{3}{2}$ (e) $\dfrac{2}{3}$

12. How many points with integral coordinates lie within the interior of the circle: $x^2 + y^2 = 25$?

(a) 55 (b) 57 (c) 60 (d) 65 (e) 69

13. If $f(x) = 3x^2 - 5x + 6$, what is the solution of $f(x + 1) = f(x - 1)$?

(a) $\dfrac{1}{2}$ (b) $\dfrac{5}{6}$ (c) $\dfrac{7}{6}$ (d) $-\dfrac{1}{2}$ (e) $\dfrac{-5}{6}$

14. What is the sum of the solutions of the equation $\log_2(x^2 + 6x + 1) = 3$?

(a) -3 (b) -4 (c) -5 (d) -6 (e) -7

15. What is the solution of $x^2 + 4x - 12 \leq 0$?

(a) $-6 \leq x \leq 2$ (b) $-4 \leq x \leq 3$

(c) $-2 \leq x \leq 6$ (d) $x \leq -4$ or $x \geq 3$

(e) $x \leq -6$ or $x \geq 2$

16. What is $\dfrac{8x^3 - 27}{8x^2 + 12x + 18} \times \dfrac{15x^2 + 10x}{6x^2 - 5x - 6}$?

(a) $\dfrac{2x - 3}{2x + 3}$ (b) $\dfrac{10x - 15}{2x + 3}$

(c) $\dfrac{10x^2 - 15x}{2x + 3}$ (d) $\dfrac{10x^2 + 15x}{2x - 3}$

(e) $\dfrac{5x}{2}$

17. Find a polynomial, $P(x)$, of lowest degree with real coefficients that has as zeros i and $1 - i$ and $P(1) = 4$.

(a) $2x^4 - 4x^3 - 6x^2 + 5x + 7$ (b) $2x^4 - 4x^3 + 6x^2 + 4x - 4$

(c) $2x^4 - 4x^3 - 6x^2 + 4x + 8$ (d) $2x^4 + 4x^3 - 6x^2 + 5x - 1$

(e) $2x^4 - 4x^3 + 6x^2 - 4x + 4$

18. What is the distance from the vertex of $y = x^2 - 4x + 3$ to the center of $\dfrac{(x-1)^2}{4} + (y+2)^2 = 1$?

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{7}$ (e) 3

19. What is the equation of the circle which has as endpoints of a diameter $(-2, 3)$ and $(4, 7)$?

(a) $x^2 + y^2 - 2x - 10y = -13$ (b) $x^2 + y^2 + 2x - 10y = -13$

(c) $x^2 + y^2 - 2x + 10y = -13$ (d) $x^2 + y^2 + 2x - 10y = 13$

(e) $x^2 + y^2 - 2x + 10y = 13$

20. What is the sum of the solutions of the equation, $\sin(x) - \cos(x) = 1$, $0 \leq x \leq 2\pi$?

(a) $\frac{3\pi}{2}$ (b) 2π (c) $\frac{5\pi}{2}$ (d) 3π (e) 4π

21. $\sec(x)\csc(x) - \cot(x)$ equals which of the following?

(a) $\cos(x)$ (b) $\csc(x)$ (c) $\cot(x)$ (d) $\tan(x)$ (e) $\sec(x)$

22. If $\sin(t) = \frac{1}{3}$, $\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$, and $\cos(x) = \frac{3}{4}$, $\frac{3\pi}{2} \leq x \leq 2\pi$, what is $\sin(t - x)$?

(a) $\frac{3 - 2\sqrt{14}}{12}$ (b) $\frac{3 + 2\sqrt{14}}{12}$ (c) $\frac{-3 + 2\sqrt{14}}{12}$ (d) $\frac{-3 - 2\sqrt{14}}{12}$

(e) $\frac{2 + 3\sqrt{14}}{12}$

23. Two five-digit numbers, both multiples of 9, are given as $12M48$ and $2344N$, where M and N are positive integers. What is $M + N$?

(a) 4 (b) 5 (c) 6 (d) 7 (e) 8

24. If $x - y = 5$ and $x^3 - y^3 = 100$, what is xy?

(a) -3 (b) $-\frac{5}{3}$ (c) $-\frac{10}{3}$ (d) 3 (e) $\frac{5}{3}$

25. Simplify $\sqrt{\frac{1}{25} + \frac{1}{144}}$.

(a) $\frac{1}{5} + \frac{1}{12}$ (b) $\frac{17}{60}$ (c) $-\frac{17}{60}$ (d) $\frac{2}{17}$ (e) $\frac{13}{60}$

26. Which of the following is the least positive integer n for which $n^{12} > 13^8$?

(a) 4 (b) 5 (c) 6 (d) 8 (e) 9

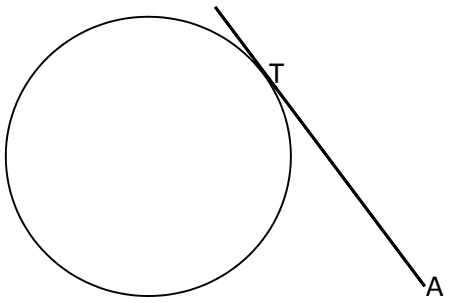
27. How many points of intersection do the graphs of $y = 2x^3 - x^2 + 10x - 11$ and $y = 2x^2 - 2x + 7$ have?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

28. Which one of the following is largest?

(a) $\sin(50^\circ)$ (b) $\tan(50^\circ)$ (c) $\cos(50^\circ)$ (d) $\csc(50^\circ)$ (e) $\sec(50^\circ)$

29. The equation $x^2 + y^2 - 4x + 6y + 12 = 0$ represents a circle. From the point A with coordinates $(-1, 2)$ draw a line tangent to this circle at a point T. Find the length of the line segment AT.



(a) 7 (b) 6 (c) $\sqrt{12}$ (d) $\sqrt{33}$

(e) The tangent line does not exist.

30. For the $\triangle ABC$, the lengths of the sides are $AB = AC = 5$ and $BC = 2$. Let D be a point on the line segment AC such that $AC \perp BD$. Find the length BD.

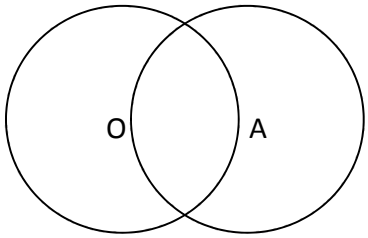
(a) $\frac{4}{5}\sqrt{6}$ (b) $\frac{2}{5}\sqrt{6}$ (c) $\frac{12}{5}$ (d) 3

(e) It can not be determined.

31. How many odd integers between 1000 and 9999 have distinct digits?

(a) 2246 (b) 2245 (c) 2242 (d) 2240 (e) 2235

32. Two circles of radius 2 pass through the centers of each other. What is the area of the common part of the two circles?



(a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}+\frac{\sqrt{3}}{2}$ (c) $2\pi-\sqrt{2}$ (d) $\frac{8\pi}{3}-2\sqrt{3}$
(e) $\frac{8\pi}{3}$

33. Suppose $\triangle ABC$ is a right triangle with side lengths $AB=3$, $BC=4$ and $AC=5$. Let P, Q and R be points on line segments BC, CA and AB respectively, with the property $QP \perp BC$ and $QR \perp AB$. Find the value $QR+\frac{4}{3}QP$.

(a) 3 (b) 4 (c) 5 (d) 6
(e) It can not be determined.

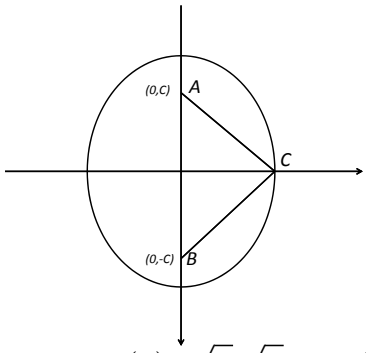
34. What is the period of $\cos^2(x+1)-\sin^2(x+1)$?

(a) 1 (b) $\frac{\pi}{2}$ (c) π (d) 2π (e) 4π

35. What is the number of solutions to $\sin^3(\theta)+\cos^3(\theta)=0$, $0\le\theta\le 2\pi$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

36. The isosceles triangle ABC shown has vertices at either focus of the ellipse : $\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$ and the third vertex is at the point where the ellipse meets the X-axis. What is the area of the triangle ABC?



(a) $\dfrac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\sqrt{2}\sqrt{3}$ (d) $2\sqrt{3}$ (e) $4\sqrt{3}$

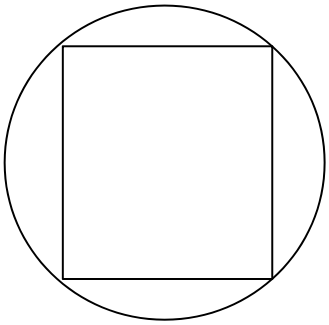
37. Write $\displaystyle\sum_{n=1}^{2011} i^n$ in the form of $a + bi$, where $i = \sqrt{-1}$.

(a) 1 (b) -1 (c) i (d) $-i$ (e) $1 + i$

38. The number 14,400,000 can be factored as $14,400,000 = 2^9 3^2 5^5$. How many perfect square numbers evenly divide into 14,400,000?

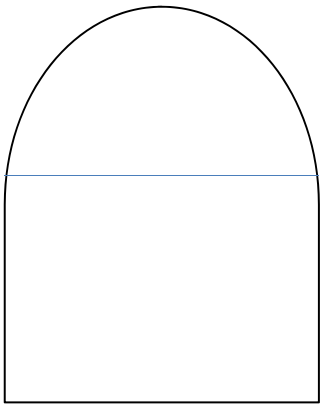
(a) 12 (b) 16 (c) 24 (d) 30 (e) 144

39. A square is inscribed in a circle as shown. If the area of the circle is 10 times its circumference, what is the area of the square?



(a) 400 (b) $400\sqrt{2}$ (c) $600\sqrt{2}$ (d) 700 (e) 800

40. A window is in the shape of a square surmounted by a semicircle as shown. If the diagonal of the square is $10\sqrt{2}$ inches, then what is the total area of the window?



(a) $100 + 25\pi$ (b) $100 + 50\pi$ (c) $\dfrac{200 + 25\pi}{2}$ (d) $200 + 25\pi$
(e) $200 + 50\pi$