

**Question 1.** A piece of string is 40 centimeters long. It is cut into three pieces. The longest piece is 3 times as long as the middle-sized piece and the shortest piece is 23 centimeters shorter than the longest piece. Find the length of the longest piece in centimeters.

**Answer:** 27

**Question 2.** If  $(x, y)$  is a solution of  $3^2 + 4^2 + 6^2 + 8^2 = x^2 + y^2$ , where  $x$  and  $y$  are positive integers, find  $x + y$ .

**Answer:** 13 or 15

**Question 3.** If  $\sin(\theta) = \frac{1}{3}$ , find  $\tan^2(\theta)$ .

**Answer:**  $\frac{1}{8}$

**Question 4.** What is the sum of the solutions of the equations  $x^3 + x^2 - 4x - 4 = 0$  and  $|x^2 + 2x| = 3$ ?

**Answer:** -5

**Question 5.** If  $x + y = -6$  and  $x^3 + y^3 = -60$ , what is  $xy$ ?

**Answer:**  $\frac{26}{3}$

**Question 6.** Evaluate  $(\log_3 27)^{\log_9 64}$ .

**Answer:** 8

**Question 7.** Suppose  $f(x) = ax + b$ , where  $a$  and  $b$  are real numbers and  $f(f(f(x))) = 27x + 39$ . Find the value of  $a + b$ .

**Answer:** 6

**Question 8.** Find the volume of a cube whose surface area is four times the number of its faces.

**Answer:** 8

**Question 9.** The average of 5 numbers is 28. What would a 6th number have to be to bring the average up to 30?

**Answer:** 40

**Question 10.** Which is the smaller of these four:  $(2^4)^8$  or  $(8^2)^4$  or  $(4^8)^2$  or  $(2^8)^4$ ?

**Answer:**  $(8^2)^4$

**Question 11.** If  $\sec \theta = \frac{3}{2}, \pi \leq \theta \leq 2\pi$ , then what is  $\tan \theta + \cot \theta$ ? Rationalize the denominator.

**Answer:**  $-\frac{9\sqrt{5}}{10}$

**Question 12.** What is the sum:  $\frac{1}{1 - \frac{1}{1 - \frac{1}{x}}} + \frac{1}{1 - \frac{1}{1 + \frac{1}{x}}}$ ?

**Answer:** 2

**Question 13.** Find the sum:  $i - i^2 + i^3 - i^4 + \dots + i^{2011} - i^{2012}$ , where  $i = \sqrt{-1}$ .

**Answer:** 0

**Question 14.** The average grade of a class on a 100-point exam is 89. The teacher gives an extra 10 points to one of the students and 3 extra points to each of the other students. The new average is now 92.5. How many students are in the class?

**Answer:** 14

**Question 15.** Suppose  $f(n) = f(n+1) - f(n-1)$  for  $n = 0, 1, 2, \dots$  and  $f(1) = 0, f(3) = 1$ . Find the value of  $f(2) + f(4)$ .

**Answer:** 3

**Question 16.** Given a bag with 4 pairs of white gloves, 5 pairs of red gloves and 6 pairs of blue gloves, find the number of gloves one needs to pull out of the bag one glove at a time to ensure a pair of matching gloves, where there are both right-handed and left-handed gloves.

**Answer:** 16