Question 1. A piece of string is 40 centimeters long. It is cut into three pieces. The longest piece is 3 times as long as the middle-sized piece and the shortest piece is 23 centimeters shorter than the longest piece. Find the length of the longest piece in centimeters.
Answer: 27

Question 2. If \((x, y)\) is a solution of \(3^2 + 4^2 + 6^2 + 8^2 = x^2 + y^2\), where \(x\) and \(y\) are positive integers, find \(x + y\).
Answer: 13 or 15

Question 3. If \(\sin(\theta) = \frac{1}{3}\), find \(\tan^2(\theta)\).
Answer: \(\frac{1}{8}\)

Question 4. What is the sum of the solutions of the equations \(x^3 + x^2 - 4x - 4 = 0\) and \(|x^2 + 2x| = 3\)?
Answer: -5

Question 5. If \(x + y = -6\) and \(x^3 + y^3 = -60\), what is \(xy\)?
Answer: \(\frac{26}{3}\)

Question 6. Evaluate \((\log_3 27)^{\log_9 64}\).
Answer: 8

Question 7. Suppose \(f(x) = ax + b\), where \(a\) and \(b\) are real numbers and \(f(f(f(x))) = 27x + 39\). Find the value of \(a + b\).
Answer: 6

Question 8. Find the volume of a cube whose surface area is four times the number of its faces.
Answer: 8
Question 9. The average of 5 numbers is 28. What would a 6th number have to be to bring the average up to 30?
Answer: 40

Question 10. Which is the smaller of these four: \((2^4)^8\) or \((8^2)^4\) or \((4^8)^2\) or \((2^8)^4\)?
Answer: \((8^2)^4\)

Question 11. If \(\sec \theta = \frac{3}{2}, \pi \leq \theta \leq 2\pi\), then what is \(\tan \theta + \cot \theta\)? Rationalize the denominator.
Answer: \(-\frac{9\sqrt{5}}{10}\)

Question 12. What is the sum: \(\frac{1}{1-\frac{1}{x}} + \frac{1}{1-\frac{1}{1+x}}\)?
Answer: 2

Question 13. Find the sum: \(i - i^2 + i^3 - i^4 + ... + i^{2011} - i^{2012}\), where \(i = \sqrt{-1}\).
Answer: 0

Question 14. The average grade of a class on a 100-point exam is 89. The teacher gives an extra 10 points to one of the students and 3 extra points to each of the other students. The new average is now 92.5. How many students are in the class?
Answer: 14

Question 15. Suppose \(f(n) = f(n+1) - f(n-1)\) for \(n = 0, 1, 2, ...\) and \(f(1) = 0, f(3) = 1\). Find the value of \(f(2) + f(4)\).
Answer: 3

Question 16. Given a bag with 4 pairs of white gloves, 5 pairs of red gloves and 6 pairs of blue gloves, find the number of gloves one needs to pull out of the bag one glove at a time to ensure a pair of matching gloves, where there are both right-handed and left-handed gloves.
Answer: 16