

Georgia Southwestern State University
Mathematics Tournament
Test Booklet
2012

INSTRUCTIONS: This is a 90-minute, 40-problem, multiple-choice exam. There are five (5) possible responses to each question. You are to select the one best answer to each question. You may mark on the test booklet and use the back of each page for additional work. When you are sure of your answer, circle the letter of the choice you have made in the test booklet. After you have worked all problems you can work, transfer your answers to the score sheet which has your student number encoded. Darken completely the blank below the letter of your response to each question with a No. 2 pencil provided by GSW. If you decide to change your answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark below row 40. Your score will be computed by the formula $40 + (4C - I)$, where C is the number of correct answers and I is the number of incorrect answers. If you can definitely rule out at least one choice it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. You may not use a calculator on this test.

Review and check your score sheet carefully. Your student identification number has been encoded on your red and white score sheet and has been checked by our marked sense card reader. This number is in the "I.D. Number" section at the top; if you alter this number in any way, you may **disqualify yourself and your team from consideration for any awards.**

When you complete your test, bring your answer sheet to the Test Monitor. You may keep your pencil and test booklet. You may leave the exam room after you have handed in your answer sheet.

PLEASE DO NOT OPEN UNTIL INSTRUCTED TO DO SO

1. Farmer Cliett sold 20 goats for \$200 each in January and 30 goats for \$300 each in February. What is the average price for each of the goats Farmer Cliett sold during these two months?
- (a) \$240 (b) \$250 (c) \$260 (d) \$270 (e) \$280
2. In this exam, suppose someone answers all 40 questions and receives a score of 80. How many questions does the person answer incorrectly? (Read the instructions carefully.)
- (a) 8 (b) 12 (c) 16 (d) 20 (e) 24
3. What is the number of positive integers whose square is a factor of 10,000?
- (a) 4 (b) 5 (c) 8 (d) 9 (e) 10
4. A stadium has a capacity of 100,000 and has 5 gates and at each of the 5 gates a maximum of 400 people can enter per minute. What is the shortest amount of time in minutes the stadium will take to reach capacity?
- (a) 30 (b) 40 (c) 50 (d) 60 (e) 80
5. What is the unit digit of 2^{2012} ?
- (a) 0 (b) 2 (c) 4 (d) 6 (e) 8
6. If $\tan(\theta) = 7$, find $\frac{57}{\cot(\theta) + \cos^2(\theta)}$.
- (a) 0 (b) $\sqrt{50}$
(c) 150 (d) $\sqrt{150}$
(e) 350

7. $\cos^2 \theta + \sec^2 \theta \cot^2 \theta \tan^2 \theta \cos^2 \theta \sin^2 \theta$ equals which of the following?

- (a) 0 (b) 1 (c) $\sec \theta$ (d) $\tan \theta$ (e) $\cot \theta$

8. The area of an ellipse is given by πab , where a is one semi-axis and b is the other. What is the area of this ellipse: $\frac{(x-3)^2}{2} + \frac{(y-4)^2}{8} = 1$?

- (a) 4π (b) $2\sqrt{2}\pi$ (c) $\frac{\pi}{4}$ (d) 16π (e) π

9. What is the equation of the parabola with focus at $(2, 8)$ and vertex at $(2, 5)$?

- (a) $y = 2x^2 - 5$ (b) $y = \frac{3}{4}x^2 - 5$
(c) $y = 4x^2$ (d) $y = 12(x-2)^2 + 5$
(e) $y = \frac{1}{12}(x-2)^2 + 5$

10. What is the equation of a tangent line to $x^2 + y^2 = 25$ where $x = -3$?

- (a) $y = 3x + 8$ (b) $y = 4x + 6$
(c) $y = \frac{3}{4}x + \frac{25}{4}$ (d) $y = \frac{4}{2}x + \frac{25}{3}$
(e) $y = \frac{4}{3}x$

11. A multimillionaire bequeaths $\frac{1}{3}$ of his estate to his wife, $\frac{1}{7}$ of the remaining amount to his only daughter, $\frac{1}{6}$ of the now remaining amount to his only son, and the final remaining \$30,000,000 to his alma mater. What was the value of his estate?

- (a) \$40,000,000 (b) \$50,000,000
(c) \$54,000,000 (d) \$60,000,000
(e) \$63,000,000

12. If $f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, where $x, y, z \neq 0$; find $f(2, 12, 9)$.
- (a) 10 (b) 8 (c) 6 (d) 4 (e) 2
13. What is the least value of y that satisfies the equation:
 $|x - 4| + |y - 3| = 10$?
- (a) 0 (b) -5 (c) -7 (d) -10 (e) -12
14. What is the sum of the two numbers that satisfy the condition: A number plus six times its reciprocal equals 5?
- (a) 1 (b) 5 (c) 7 (d) $\frac{1}{3}$ (e) $\frac{3}{4}$
15. Solve for x : $\sqrt[5]{16} = 8$.
- (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$ (e) 2
16. What is $\frac{1}{\log_2 24} + \frac{1}{\log_3 24} + \frac{1}{\log_4 24}$?
- (a) $\frac{1}{2}$ (b) 1 (c) e^{24} (d) $\ln 24$ (e) 2
17. Which of the following is equal to $\sin \theta(\cot \theta + \tan \theta)$?
- (a) $\cos \theta$ (b) $\tan \theta$ (c) $\cot \theta$ (d) $\sec \theta$ (e) $\csc \theta$
18. When $x^3 + \lambda x + 4$ is divided by $x - 2$, the remainder is 5. When $x^3 - x^2 + \mu x + 3$ is divided by $x + 2$, the remainder is -2. What is the sum $\lambda + \mu$?
- (a) -12 (b) -9 (c) 0 (d) 4 (e) -7

19. Which expression is equal to $\frac{4x^3 - 4x^2 - 3x}{16x^3 - 54} \div \frac{12x^2 - 3}{8x^2 + 12x + 18}$?

(a) $\frac{1}{3}$ (b) $\frac{2x - 3}{2x - 1}$

(c) $\frac{x(2x + 1)}{2x - 3}$ (d) $\frac{x}{6x - 3}$

(e) The correct answer is not given.

20. Write in the form of $a + ib$: $\frac{i^{2012} - i^{2013}}{1 + i}$, where $i = \sqrt{-1}$.

(a) $-i$ (b) i (c) -1 (d) 1 (e) $1 - i$

21. In an isosceles right triangle, one of the angles that is not a right angle has measure θ . What is the value of $\sin(2\theta)$?

(a) $\frac{1}{2}$ (b) $\sqrt{2}$ (c) 1 (d) $\sqrt{3}$ (e) 2

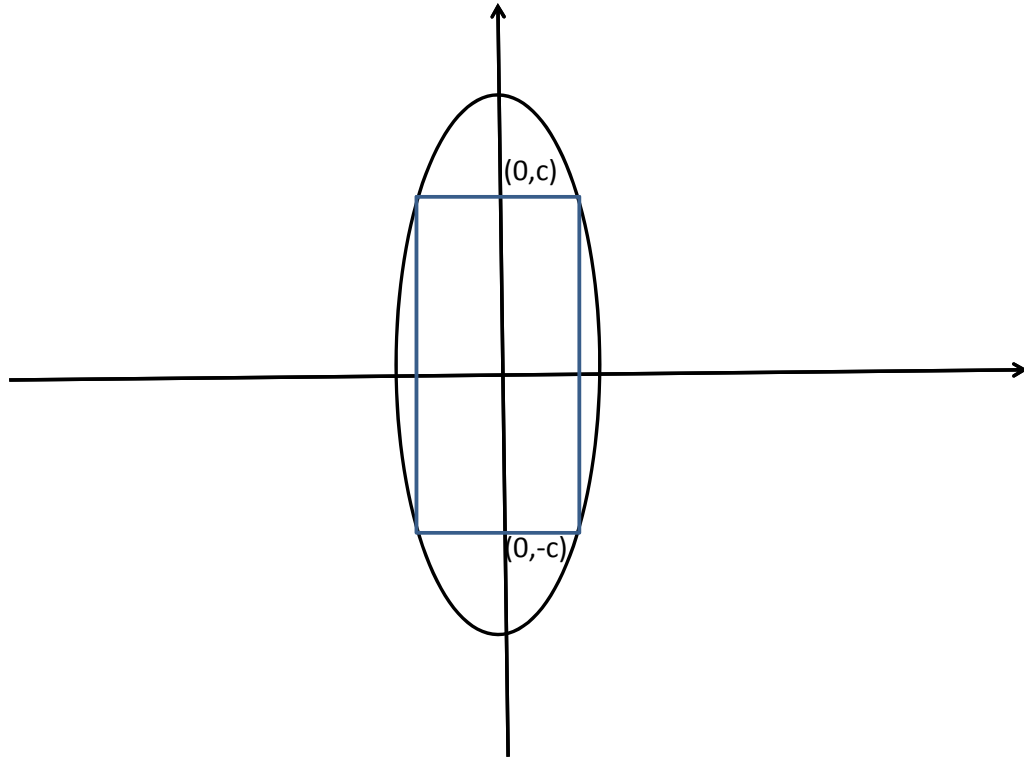
22. Simplify $\sqrt{\frac{1}{144} - \frac{1}{169}}$.

(a) $\frac{1}{12} - \frac{1}{13}$ (b) $\frac{1}{12} - \frac{1}{13}i$

(c) $-\frac{1}{13}$ (d) $\frac{3}{160}$

(e) $\frac{5}{156}$

23. A rectangle is inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ so that two sides of the rectangle pass through the foci $(0, c)$ and $(0, -c)$ as shown. What is the area of the rectangle?

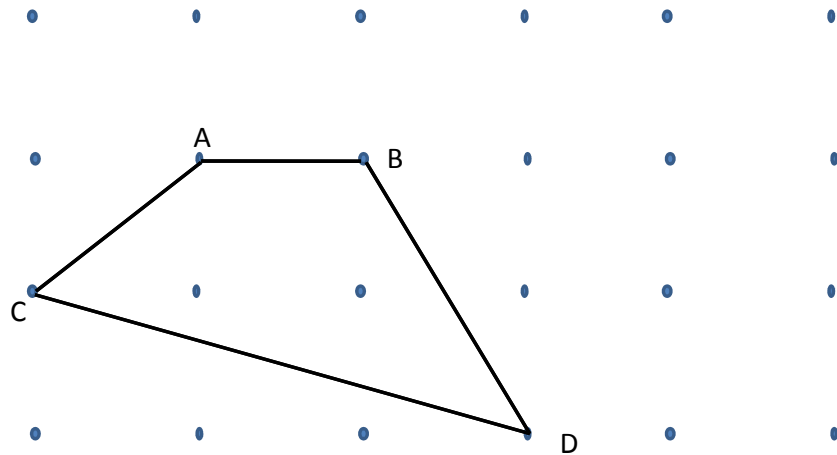


- (a) $\frac{36}{5}$ (b) $\frac{72}{5}$ (c) $\frac{144}{5}$ (d) $\frac{225}{9}$ (e) $\frac{225}{81}$
24. The area of a right isosceles triangle is 5 times the perimeter of the triangle. What is the perimeter of the triangle?
- (a) $20 + (20\sqrt{2})$ (b) $40 + (40\sqrt{2})$
(c) $60 + (40\sqrt{2})$ (d) $80 + (20\sqrt{2})$
(e) $100 + (40\sqrt{2})$

28. What is the sum of the solutions to $\sin^4(x) - \cos^4(x) = 0$, $0 \leq x \leq 2\pi$?

- (a) π (b) 2π (c) $\frac{5\pi}{2}$ (d) 4π (e) 6π

29. Any two adjacent pegs in the diagram are spaced 1 unit apart. What is the area of quadrilateral ABCD?

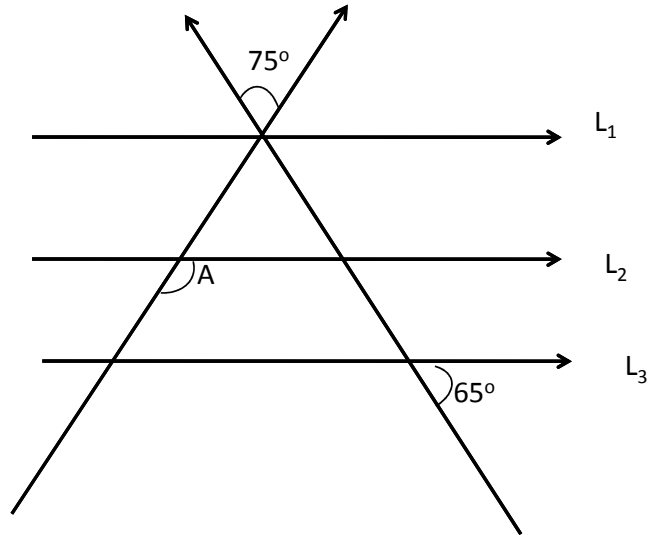


- (a) 2 (b) $\frac{3}{2}$ (c) 3 (d) $\frac{5}{2}$ (e) 4

30. If $18x^2 - 6\sqrt{2} = 27$, what is $24x^2 - 8\sqrt{2}$?

- (a) 28 (b) 30 (c) 32 (d) 36 (e) 40

31. Lines L_1 , L_2 , and L_3 are parallel. Find the measure of angle A.



- (a) 105° (b) 115° (c) 125° (d) 135° (e) 140°

32. Which of the following is the least positive integer n for which $n^{24} > 14^{16}$?

- (a) 6 (b) 7 (c) 8 (d) 9 (e) 10

33. How many points of intersection do the graphs of $y = 2x^3 + 4x^2 + 4x + 7$ and $y = x^3 + 7x^2 + 5x + 4$ have?

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

34. What is the equation of the circle concentric with $x^2 + y^2 + 4x - 6y = 12$ that passes through $(1, 3 + \sqrt{7})$?
- (a) $x^2 + y^2 + 4x - 6y = 1$ (b) $x^2 + y^2 + 4x - 6y = 2$
(c) $x^2 + y^2 + 4x - 6y = 3$ (d) $x^2 + y^2 + 4x - 6y = 4$
(e) $x^2 + y^2 + 4x - 6y = 5$
35. Which one of the following is an asymptote of $4x^2 - 9y^2 + 16x + 54y = 101$?
- (a) $3y = 2x - 12$ (b) $2y = 3x - 12$
(c) $2y = 3x + 12$ (d) $2y = 3x + 13$
(e) $3y = 2x + 13$
36. What is the exact value of $\cos\left(\frac{7\pi}{12}\right)$?
- (a) $\frac{\sqrt{2} - \sqrt{6}}{4}$ (b) $\frac{\sqrt{2} + \sqrt{6}}{4}$
(c) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (d) $\frac{-\sqrt{2} - \sqrt{6}}{4}$
(e) Correct answer not given.
37. If $P(x)$ is a cubic polynomial that has 1, -2, and 3 as zeros and $P(2) = 8$, what is $P(-1)$?
- (a) -10 (b) -12 (c) -14 (d) -16 (e) -18
38. Which of the following does $-2\sqrt[3]{375} + 5\sqrt[3]{192} - 4\sqrt[3]{81}$ equal?
- (a) $-3\sqrt[3]{102}$ (b) $-4\sqrt[3]{86}$
(c) $-6\sqrt[3]{49}$ (d) $-12\sqrt[3]{36}$
(e) $-2\sqrt[3]{3}$

39. Which one of the following lies on line L if L passes through $(-2,-6)$ and $(4,18)$?

(a) $(-98,-388)$

(b) $(-76,-308)$

(c) $(55,222)$

(d) $(72,294)$

(e) $(88,352)$

40. Suppose that the probability that a baby is a girl is $\frac{1}{2}$ and the probability that a baby is a boy is $\frac{1}{2}$. What is the probability that a family of four children has two boys and two girls in four independent births?

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{1}{4}$

(d) $\frac{3}{8}$

(e) $\frac{3}{10}$