Georgia Southwestern State University
Mathematics Tournament
Test Booklet
2013

INSTRUCTIONS: This is a 90-minute, 40-problem, multiple-choice exam. There are five (5) possible responses to each question. You are to select the one best answer to each question. You may mark on the test booklet and use the back of each page for additional work. When you are sure of your answer, circle the letter of the choice you have made in the test booklet. After you have worked all problems you can work, transfer your answers to the score sheet which has your student number encoded. Darken completely the blank below the letter of your response to each question with a No. 2 pencil provided by GSW. If you decide to change your answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark below row 40. Your score will be computed by the formula $40 + (4C - I)$, where C is the number of correct answers and I is the number of incorrect answers. If you can definitely rule out at least one choice it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. You may not use a calculator on this test.

Review and check your score sheet carefully. Your student identification number has been encoded on your red and white score sheet and has been checked by our marked sense card reader. This number is in the "I.D. Number" section at the top; if you alter this number in any way, you may **disqualify yourself and your team from consideration for any awards**.

When you complete your test, bring your answer sheet to the Test Monitor. You may keep your pencil and test booklet. You may leave the exam room after you have handed in your answer sheet.

PLEASE DO NOT OPEN UNTIL INSTRUCTED TO DO SO

1. Farmer Kipp sold 25 horses for a total of \$7500 in January and 25 horses for a total of \$10500 in February. What is the average price of the horses Farmer Kipp sold during these two months?

   (a) \$340 (b) \$350 (c) \$360 (d) \$370 (e) \$380

2. A used car dealer buys a car from another dealer and marks it up 40%. He then sells the car for \$8400. How much profit did he make?

   (a) \$2000 (b) \$2400 (c) \$2800 (d) \$3200 (e) \$3400

3. Which has the largest value?

   (a) $\sin(30^o)$ (b) $\cos(30^o)$ (c) $\tan(30^o)$ (d) $\cot(30^o)$ (e) $\sec(30^o)$

4. Express $\dfrac{2x^2-x-6}{4x^2-9} \div \dfrac{x^3-8}{6x^2-x-12}$ as a fraction in lowest terms.

   (a) $\dfrac{2x-3}{x^2+2x+4}$ (b) $\dfrac{2x-3}{x^2-2x+4}$

   (c) $\dfrac{3x+4}{x^2-2x+4}$ (d) $\dfrac{3x-4}{x^2+2x+4}$

   (e) $\dfrac{3x+4}{x^2+2x+4}$

5. If $\dfrac{\sqrt{x^3y^{-5}z^4k^{-12}}\sqrt[3]{x^{30}y^{-21}z^6k^{15}}}{\sqrt{x^{11}y^{-23}z^{-12}k^{10}}}$ is expressed in the form of $x^my^az^tk^h$, what is $m+a+t+h$?

   (a) 10 (b) 11 (c) 12 (d) 13 (e) 14

6. What is the remainder when $x^4-2x^3+3x^2-4x-5$ is divided by x^2+x-2?

   (a) $18x-11$ (b) $-18x+11$

   (c) $-2x-21$ (d) $-2x+21$

   (e) $2x+11$

7. Find $P(-2)$ if $P(x)$ is a polynomial of lowest degree with real coefficients with zeros of -1 and $2i$ and $P(0) = -12$.

   (a) 18 (b) 20 (c) 22 (d) 24 (e) 26

8. Which one of the following is the directrix of $y = \dfrac{x^2 - 4x + 4}{32}$?

   (a) $y = -3$ (b) $y = -4$ (c) $y = -5$ (d) $y = -6$ (e) $y = -8$

9. Which one of the following is a focus of $4x^2 + 9y^2 - 8x + 72y = -112$?

   (a) $(1, -4 - \sqrt{5})$ (b) $(1, -4 - \sqrt{3})$
   (c) $(1 + \sqrt{13}, -4)$ (d) $(1 - \sqrt{3}, -4)$
   (e) $(1 - \sqrt{5}, -4)$

10. Which of the following is equal to $\sin\left(\arccos\left(\dfrac{1}{3}\right) - \arcsin\left(\dfrac{1}{2}\right)\right)$?

    (a) $\dfrac{2\sqrt{6} - 1}{6}$ (b) $\dfrac{4\sqrt{3} - 1}{6}$
    (c) $\dfrac{2\sqrt{6} + 1}{6}$ (d) $\dfrac{1 - 4\sqrt{3}}{6}$
    (e) $\dfrac{1 - 2\sqrt{6}}{6}$

11. Find the sum of all values of $x, 0 \leq x \leq 2\pi$, that satisfy the equation $2\cos^2(x) + \cos(x) - 1 = 0$.

    (a) $\dfrac{3\pi}{2}$ (b) 2π (c) $\dfrac{5\pi}{2}$ (d) 3π (e) $\dfrac{7\pi}{2}$

12. If $3^{(x+1)} = 81$ and $2^{(4y-1)} = 32$, then $x^y =$?

    (a) 5 (b) 26 (c) $\sqrt{26}$ (d) 27 (e) $\sqrt{27}$

13. What is the y-coordinate of the rightmost point of intersection of the line $x - y = 6$ and the parabola $y = x^2 - 3x - 18$?

(a) 0 (b) -2 (c) -8 (d) -6 (e) -10

14. If $x + y = 6$ and $3x - 2y = 4$, then what is $6x + y$?

(a) 12 (b) 14 (c) 15 (d) 20 (e) 22

15. If the area of an ellipse is given as πab, where a is the semi-major axis and b is the semi-minor axis, what is the area of the ellipse $9x^2 + 16y^2 = 25$?

(a) $\frac{16\pi}{25}$ (b) $\frac{12\pi}{25}$ (c) $\frac{25\pi}{9}$ (d) $\frac{25\pi}{12}$ (e) $\frac{9\pi}{25}$

16. What is the area of a square inscribed in a circle of radius 25?

(a) 500 (b) 900 (c) 1250 (d) 1600 (e) 2500

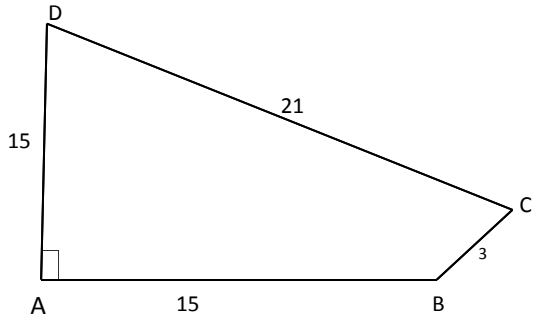
17. What is the perimeter of an isosceles trapezoid with height 8 inches, bases that differ by 12 inches, and an area of 96 square inches?

(a) 36 (b) 38 (c) 40 (d) 42 (e) 44

18. Write in the form of $a + ib$: $\frac{i^{2012} - i^{2013}}{1 + i}$, where $i = \sqrt{-1}$.

(a) $-i$ (b) i (c) -1 (d) 1 (e) $1 - i$

19. Find the area of quadrilateral ABCD.



(a) 105 (b) 144

(c) $15(\sqrt{3}+\sqrt{7})$ (d) 196

(e) $225-15\sqrt{3}$

20. A positive integer has 444 digits all of which are 4s. What is the remainder when this number is divided by 444?

(a) 0 (b) 4 (c) 40 (d) 44 (e) 440

21. What is
$1(1\times 2)\left(\frac{1}{1}-\frac{1}{2}\right)+2(2\times 3)\left(\frac{1}{2}-\frac{1}{3}\right)+...+99(99\times 100)\left(\frac{1}{99}-\frac{1}{100}\right)$?

(a) 100 (b) 2350 (c) 4050 (d) 4950 (e) 5050

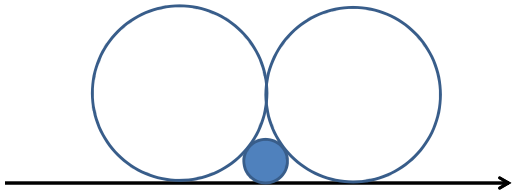
22. Eight equally sized balls fit snugly into a cubical box. What part of the box is occupied by the balls?

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{8}$ (e) $\frac{\pi}{4}$

23. When a fair coin is flipped eight times, find $\log_2(p)$, where p is the probability that no two consecutive flips have the same outcome.

(a) -3 (b) -7 (c) $\frac{2^8-2^7}{2^8}$ (d) $\frac{1}{16}$ (e) 0

24. All three circles are tangent to the given line and to each other in the following picture. The larger two circles are congruent. If the small circle has a radius of 10, what is the radius of either of the bigger circles?

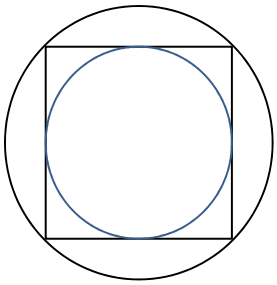


(a) 20 (b) $20\sqrt{2}$ (c) 30 (d) 40 (e) $\dfrac{100}{\sqrt{2}}$

25. What is the sum of the solutions $x, 0 \leq x \leq 2\pi$ to $\sec^{201}(x) + \csc^{201}(x) = 0$?

(a) π (b) 2π (c) $\dfrac{5\pi}{2}$ (d) 3π (e) 5π

26. A small circle is inscribed in a square and the square in turn is inscribed in a larger circle, as shown. What is the difference in the areas of the two circles if the diameter of the inside circle is 1?

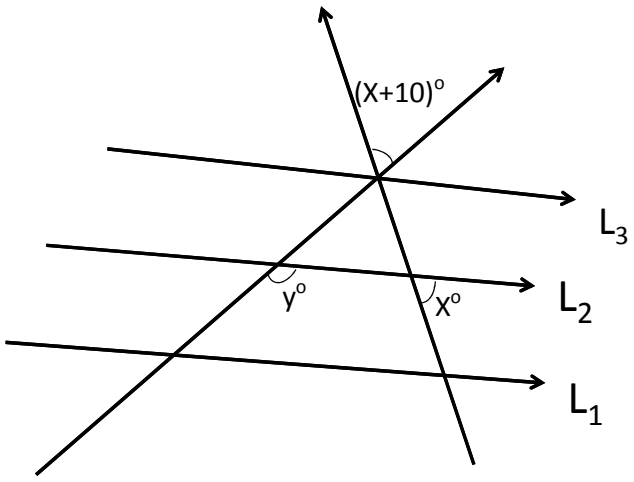


(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{\sqrt{2}}$ (d) $\dfrac{3\pi}{4}$ (e) $\dfrac{7\pi}{4}$

27. For any positive integer n, find the value of $(-1)^{n^{4n}+2n^2+n+1}$.

(a) -1 (b) 1 (c) $(-1)^n$ (d) 0 (e) $(-1)^{n+1}$

28. In the diagrams, lines L_1, L_2 and L_3 are parallel. What is the measure of angle Y?



(a) $(180 - X)^o$ (b) $(2X + 10)^o$

(c) $(170 - X)^o$ (d) $(80 + X)^o$

(e) $(190 - X)^o$

29. Compute:
$\sin^2(13^o) + \cos^2(13^o) + \sec^2(13^o) + \csc^2(13^o) - \tan^2(13^o) - \cot^2(13^o)$.

(a) -1 (b) 1 (c) 2 (d) 3 (e) $1 - \frac{\pi}{2}$

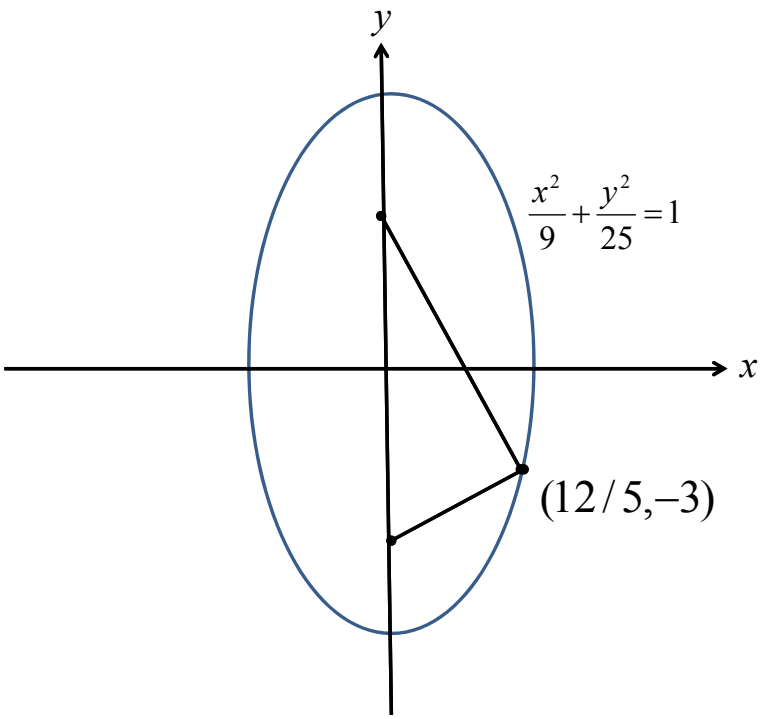
30. A function g has properties $g(x + y) = g(x) + g(y)$ for every real number x and y. Find $g(26)$ if $g(2) = 1$ and $g(16) = 12$.

(a) 12 (b) 17 (c) 21 (d) 24 (e) 28

31. Find $\ln(\tan 2^o) + \ln(\tan 4^o) + \ln(\tan 6^o) + ... + \ln(\tan 84^o) + \ln(\tan 86^o) + \ln(\tan 88^o)$.

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$ (e) π

32. Consider the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$. What is the perimeter of the triangle shown, whose vertices are the point $P\left(\dfrac{12}{5}, -3\right)$ and the two foci of the ellipse?



(a) 24

(b) $\dfrac{96}{5}$

(c) $25 - 3\sqrt{2}$

(d) $12\sqrt{2}$

(e) 18

33. Which one of the following is an asymptote of $9x^2 - 16y^2 + 36x + 96y = 252$?

(a) $4y = 3x + 18$

(b) $4y = -3x + 18$

(c) $3y = 4x + 6$

(d) $3y = -4x + 6$

(e) $4y = 3x + 6$

34. A parabola has focus $\left(\frac{1}{2}, -2\right)$ and directrix $y = -\frac{5}{2}$. Find the area of the triangle with vertices at the vertex of the parabola and the x-intercepts of the parabola.

(a) $\frac{81}{16}$ (b) $\frac{27}{8}$ (c) $\frac{9}{16}$ (d) $\frac{3}{4}$ (e) 3

35. If $\log a = 3 - \log b$, then which of the following is equal to ab?

(a) 3 (b) 1000 (c) $\log 3$ (d) $(ab)^3$ (e) $\log_3(1000)$

36. For any positive integer n, its factorial $n! = n(n-1)(n-2)....(3)(2)(1)$. What is the last digit of $103! + 93! + 83! + 73! + ... + 23! + 13! + 3!$?

(a) 0 (b) 1 (c) 4 (d) 5 (e) 6

37. If a positive number x satisfies $x^2 + \frac{1}{x^2} = 2$, then find $x + \frac{1}{x}$.

(a) $\sqrt{2}$ (b) $1 + \sqrt{2}$ (c) 1 (d) 2 (e) 4

38. Two real numbers are chosen randomly between 0 and 10. What is the probability that their sum is greater than 6?

(a) 0.36 (b) 0.41 (c) 0.72 (d) 0.82 (e) 0.92

39. A fair six-sided die (numbered 1 through 6) and a fair eight-sided die (numbered 1 through 8) are rolled. If x is the number that the six-sided die lands on and y is the number that the eight-sided die lands on, what is the probability that $|x - y| \leq 2$?

(a) $\frac{3}{14}$ (b) $\frac{3}{8}$ (c) $\frac{7}{16}$ (d) $\frac{9}{16}$ (e) $\frac{9}{14}$

40. You own 15 pairs of gloves, all different, and all of the gloves are individually jumbled in a drawer. How many gloves must you pull out to guarantee having a matching pair without looking in the drawer?

(a) 2 (b) 7 (c) 15 (d) 16 (e) 30